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Polymer Diffusion in Quenched Disorder: A Renormalization Group Approach

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We study the diffusion of polymers through quenched short-range correlated random media by renormalization group (RG) methods, which allow us to derive universal predictions in the limit of long chains and weak disorder. We take local quenched random potentials with second moment v and the excluded-volume interaction u of the chain segments into account. We show that our model contains the relevant features of polymer diffusion in random media in the RG sense if we focus on the local entropic effects rather than on the topological constraints of a quenched random medium. The dynamic generating functional and the general structure of its perturbation expansion in u and v are derived. The distribution functions for the center-of-mass motion and the internal modes of one chain and for the correlation of the center of mass motions of two chains are calculated to one-loop order. The results allow for sufficient cross-checks to have trust in the one-loop renormalizability of the model. The general structure as well as the one-loop results of the integrated RG flow of the parameters are discussed. Universal results can be found for the effective static interaction $w := u - v \geqslant 0$ and for small effective disorder coupling $\bar{v}(l)$ on the intermediate length scale l. As a first physical prediction from our analysis, we determine the general nonlinear scaling form of the chain diffusion constant and evaluate it explicitly as $D \propto N(l)^{-1}\, \mathscr{D}(\bar{v}(l)\, N(l)^{a})$ for $\bar{v}(l) \ll 1$.

KEY WORDS: Polymer dynamics; polymer diffusion; quenched random media; renormalizability; universality.

1. INTRODUCTION

How does a polymer diffuse through a quenched random medium? Is this question well posed, i.e., is there a universal answer independent of details? Or can we identify a class of models which have a universal behavior? Can

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we especially use renormalization group (RG) methods in some parameter range to settle these questions? These problems will be addressed in the present paper with a special emphasis on a systematic RG treatment.

Consider a polymer as a very long linear object of arbitrary chemical microstructure which is stiff only up to some microscopic length scale l and which is embedded in some homogeneous solvent. Let the system be in thermal equilibrium. If the monomers do not interact, the central limit theorem yields both the radius of gyration and the root mean square end to end distance scaling like $\sqrt{N}$ with chain length N, independent of the precise microstructure.

Let now the monomers repel each other on short ranges. Then the central limit theorem is generalized to a universal scaling law. Both radii scale like N^{ν} with $\nu = 0.588$ in space dimension $d = 3$, again independent of the precise microstructure. (For an attractive interaction, there is no universality.) This and other universal laws are verified both in experiments and in simulations. They can be derived by transforming polymer statics into a ϕ^4 field theory, which is known to be renormalizable to all orders of perturbation theory. (We will comment on some aspects of this connection in the body of the paper, as far as this is needed for comparison with our dynamic calculation. For a review on polymer statics see ref. 1.)

Let us briefly recall the idea of renormalizability: It implies that, of the many parameters describing the microscopic behavior of the model, only very few determine the behavior on large length scales. The basic step of a RG treatment is to rewrite the model on a different microscopic length scale, while physical observables are kept invariant. This can be done by integrating out the microscopic degrees of freedom up to some fixed length scale, which then becomes the new microscopic scale. The change of the effective parameters under this procedure is called their RG flow. The parameters of a renormalizable problem under RG flow approach a low-dimensional parameter submanifold $\mathcal{M}$. Identities derived in this distinguished manifold $\mathcal{M}$ are universal in the sense that every microscopic system within a certain parameter range on sufficiently large length scales first approaches the manifold $\mathcal{M}$ and then obeys the universal identities on $\mathcal{M}$. A point on $\mathcal{M}$, where the system parameters approximately reach $\mathcal{M}$, is related to the microscopic physical system by some nonuniversal constants. In the case of polymer statics in $d = 3$, $\mathcal{M}$ is spanned by only two parameters, namely the chain length and the two-segment interaction.

Given the fact that universality was proven for polymer statics, it is natural to ask about universality in more complicated polymer problems such as polymer diffusion in random media.

Consider as a next step the equilibrium properties of polymers in quenched random media. This problem has been discussed extensively and

controversially in the literature. These controversies arose in part from comparing different expectation values or different polymer models under the general question: What are the equilibrium properties of a polymer in a random medium? But without universality, different models give different answers. Exactly one universal regime could be identified so far,[2] namely that of vanishing segment concentration and of the effective segment–segment interaction being still repulsive after ensemble averaging. This universal regime is trivial in the sense that its mathematical structure is identical with that of polymer statics without disorder. That means it also can be transformed into a ϕ^4-theory, it is renormalizable to all orders, etc. It is related to the theory without random potentials by a mere shift of the strength of the two-segment interaction.

Now we aim beyond simple equilibrium properties and investigate the problem: How does a polymer diffuse through a quenched random medium? Can we find renormalizability and thus universality in some parameter range?

How should a realistic model be set up? The simplest approach to polymer dynamics is a Langevin equation, which describes the displacements of the individual segments due to the sum of local random thermal forces and a deterministic force derived from the equilibrium Hamiltonian. Neglecting furthermore the self-repulsion of the chain, this model is known as the Rouse model. It is exactly solvable. Inclusion of the excluded-volume interaction makes the problem only perturbatively treatable. It is renormalizable up to two-loop order.[3] Such a model, however, neglects the hydrodynamic modes of the solvent: The displacement of a segment should also cause a displacement of the surrounding solvent and thus result in a hydrodynamic flow field. The diffusion constant or viscosity of a model including hydrodynamics consistently could be renormalized only to one-loop order[4, 5] up to now.

For none of these dynamic models has a transformation to a renormalizable dynamic field theory been found so far, and we will point out some of the related difficulties in the paper at appropriate places. Therefore no transformation offers a short-cut, but the full dynamics has to be investigated perturbatively.

At first sight polymer diffusion in a random medium appears to pose even a harder problem than polymer diffusion in a pure solvent. Fortunately the random medium eases the problem in one respect: It should screen the hydrodynamic flow field. We conclude this in analogy to the hydrodynamic screening in a semidilute or dense solution, where the other polymer chains play the role of a random medium (ref. 6, p. 172).

Now the random medium should be specified. A truly quenched disorder would consist, e.g., of a gel or any other packing of a diffraction

column. One also could think of a semidilute or dense solution with the other polymers being treated as a random background. If the relaxation times of this "background" due to entanglements are sufficiently slow, one could consider this situation as "relatively quenched." In all these cases frequently a reptation model is used, which treats the polymer diffusion as a Langevin dynamics hindered by topological constraints. If the random medium forms connected structures of fairly uniform density in three-dimensional space, it can be mapped to an essentially regular array of topological obstacles. Reptation then describes the snakelike motion of the polymer through the array. As a result the diffusion constant is expected to scale not as N^{-1} as in a Rouse model, but as N^{-2}.

Consider in contrast a very irregular distribution of the medium. We then might have wide holes and narrow passages, which act as entropic traps or barriers.[7] Coarse graining a bit, an entropic trap can be considered as an attractive potential. Also, the medium, being locally attractive or repulsive, can contribute to an effectively fluctuating local binding energy. This suggests that one considers the effect of a local random potential. One should expect that such a potential distribution should slow the polymer down more than N^{-2}, since with growing chain length it will stay longer and longer in favorable energy valleys. Machta[8] has analyzed such a model and finds the diffusion constant in his model to be proportional to $N^{-1}\exp(-cv_0N^{\alpha})$, with v_0 the second moment of the potential distribution and $\alpha = 2 - dv$ the "specific heat" exponent of critical phenomena. If a prediction of this type can be verified, the random potential for sufficiently long polymers clearly dominates over the topological constraints, and it is justified to neglect the latter. It must be noted that no simple scaling ansatz can yield such an N dependence of the diffusion constant. Scaling would always predict a power law $D \propto N^{-x}$. (Our RG analysis in contrast will not give a simple scaling result.)

Experiments seem to be in favor of reptation for melts, but for polymer solutions seem to show a stronger decrease of the diffusion constant, as discussed in the review of Lodge *et al.*[9] on the experimental verification of the reptation picture. Melts or solutions map clearly to either a rather regular or a more random spacing of obstacles. Solutions therefore seem to require a proper treatment of the entropic traps. Both systems of course have no truly quenched disorder. One should expect the diffusion constant for such annealed disorder to give an upper bound to that in quenched disorder.

Having motivated the RG investigation of polymer diffusion in local random potentials so far, we note that surprisingly little analytical work can be found on such models.

Machta derives his prediction for the diffusion constant in a random

potential by coarse graining the system up to the polymer size, followed by a saddle point analysis. Especially the coarse graining needs reconsideration. Not only the whole chain, but also arbitrary parts of it can be trapped in an entropic trap. This effect can be treated systematically in a RG approach.

Besides Machta's, the early work of Martinez-Mekler and Moore[10] should be cited. They investigated exactly the same model as considered here. For the determination of the RG flow of the disorder coupling v they used a replica approach, but did not pursue the investigation further after not having found a stable fixed point of this flow. However, a stable fixed point is not an indispensable prerequisite for the application of the RG. Finding back the replica result in our fully dynamical calculation, we still can exploit the fact, that the system is driven toward the distinguished manifold $\mathcal{M}$ and evaluate the RG flow in $\mathcal{M}$. Functional relations of the form of "nonlinear" scaling laws still hold and can be evaluated perturbatively for small renormalized coupling. We will see that Machta's result reproduces the lowest order of the functional form found by our methods.

But before calculating the diffusion constant, we will investigate the theory thoroughly with a special emphasis on renormalizability. Our aim is a systematic treatment of a Langevin polymer dynamics with excluded-volume interaction and with a quenched short ranged random potential by methods of dynamic functionals and RG analysis. A systematic approach is all the more desirable, since no model for polymer dynamics so far could be proven to be renormalizable to all orders of perturbation theory. The renormalizability assumption therefore needs to undergo a broad test. At the same time the dynamic calculation also allows us to test the replica approach.

More precisely, we will proceed in the following steps: In Section 2 we recall some basics on polymer statics and its relation to a renormalizable ϕ^4-theory, we briefly discuss polymer statics with disorder, and we introduce our dynamic model.

Section 3 is devoted to the systematic setup of the tool kit of the further calculation: the dynamic generating functional is derived, the free theory is diagonalized, and the general structure of the perturbation expansion is provided. Finally the continuous chain limit as a tool for facilitating the ε-expansion ($\varepsilon = 4 - d$) and the discussion of canonical dimensions is performed and the relevance or irrelevance of various parameters in the RG sense is discussed. Though a dynamic functional approach was used earlier for polymer dynamics,[5] there is no literature we could refer to for the setup of the calculation as needed in later sections.

In Sections 4–6 three different distribution functions are discussed and analyzed. The center-of-mass motion and the internal modes are motivated

as quantities of physical interest, while the center-of-mass correlation of two chains plays a central role in the RG analysis. The physical properties of all three distribution functions are discussed and they are analyzed to one-loop order with technical details banned to some appendices. Our results allow for sufficient cross-checks to prove the one-loop renormalizability of the model not strictly, but convincingly. (A strict proof would require an abstract analysis of any generating functional, while we in fact have analyzed functionals with specific generating fields, which we believe to be representative.)

This allows us to analyze the RG flow of the parameters in Section 7. Both the one-loop results and the general structure of a renormalizable theory are discussed. The RG flow of the parameters is integrated. We finish by calculating the diffusion constant as the long-time asymptote of the center-of-mass motion. Preliminary results on both the long- as well as the short-time asymptotics of the center of mass motion can be found in ref. 11. The detailed analysis of the short-time asymptotics in comparison with Monte Carlo simulations is given in a forthcoming paper.[12]

2. DEFINITION AND DISCUSSION OF THE MODEL

In the present section we recall some features of polymer statics without and with disorder which are basic for the following analysis, including the transformation to ϕ^4-theory. We also introduce the dynamic model.

2.1. A Reminder on Polymer Statics and Renormalizability

Consider polymers consisting of $N \gg 1$ monomers in the dilute limit, i.e., at such a dilution that they can be treated as isolated chains floating in the solvent. For dimensions $2 < d < 4$, temperature $T \geqslant \Theta$, and on scales much larger than the monomer size, there are only two features of the model relevant for the calculation of properties on large scales, namely the linear chain structure with given chain length N and an effective short-ranged repulsive interaction u between the monomers. It is most convenient for calculational purposes to represent these relevant features by the Boltzmann weight $\exp(-\mathscr{H}[\mathbf{r}])$ derived from the following Hamiltonian:

$$\mathscr{H}_0[\mathbf{r}] + \mathscr{H}_u[\mathbf{r}] = \sum_{i=2}^{N} \frac{(\mathrm{r}_i - \mathrm{r}_{i-1})^2}{4l^2} + ul^d \sum_{1 \leqslant i < j \leqslant N} \delta^d(\mathrm{r}_i - \mathrm{r}_j), \qquad u \geqslant 0 \quad (2.1)$$

Here and in the rest of the paper the following notation is chosen: $r_{i\mu}$ and p_μ with $\mu = 1,..., d$ denote a component of the d-dimensional vectors r_i and

p, respectively; $\mathbf{r}$ denotes the $(d \times N)$-dimensional vector of a polymer configuration. We have

$$\begin{gathered} \mathrm{r}_i = (r_{i\mu})_{\mu=1,\ldots,d} \in \mathbb{R}^d, \qquad \mathrm{p} = (p_\mu)_{\mu=1,\ldots,d} \in \mathbb{R}^d \\ \mathbf{r} = (\mathrm{r}_i)_{i=1,\ldots,N} \in \mathbb{R}^{d \times N} \end{gathered} \tag{2.2}$$

$i = 1,\ldots, N$ defines the intrinsic metric of the chain.

That the ansatz (2.1) for $u = 0$ contains the relevant part of the chain structure can be seen easily: The distance distribution of the segments on a slightly coarse-grained level (counting, e.g., only every twentieth monomer) should be approximately Gaussian. But a Gaussian distribution is exactly the result of exponentiating $\mathscr{H}_0[\mathbf{r}]$. Also, the δ-function in $\mathscr{H}_u[\mathbf{r}]$ represents the relevant part of the short-ranged interaction. Accordingly, writing it into the exponential is a symbolic notation. The δ-function therefore should be read as either a smoothened exponentially decaying function or as $\exp[-ul^d\delta^d(\mathrm{r})] = 1 - ul^d\delta^d(\mathrm{r})$ in a virial expansion.

In the continuous chain limit $l \to 0$, $N \to \infty$, $S = Nl^2$ fixed, the model (2.1) becomes identical with the Edwards-Hamiltonian. One then needs to pay additional attention to keeping $\mathscr{H}_u[\mathbf{r}]$ well defined.

The model (2.1) is already on the critical manifold as defined in the introduction, because the length scale l of the description is already chosen so large, that irrelevant contributions like the stiffness on short scales or a three-segment interaction are eliminated. The length scale can be increased further by integration over the degrees of freedom up to the scale l'. The new model is again characterized by only the two parameters N and u, now adjusted to the new length scale l'. Starting from any $u > 0$, u asymptotically reaches a stable fixed point for sufficiently large l'. At a fixed point of u linear scaling laws can be derived, such as the radius of gyration scaling like N^ν, etc.

Clearly the very structure of renormalization implies that the change of the model parameters under a change of l must be independent of the physical quantity considered. The joint structure of (1) the parameters absorbing the same microstructure under RG flow in all expectation values, and (2) the models reaching a low-dimensional critical manifold under RG flow, is called renormalizability. Note that the existence of a fixed point within the critical manifold is not required.

That the equilibrium properties of effectively repulsive polymers in dilute solution are renormalizable to all orders of perturbation theory and therefore universal on large length scales can be proven by transforming the model (2.1) to field theory. A Laplace transformation over the chain length N maps the Green's function of the model (2.1) for a single chain to

the Green's function of a ϕ^4 theory. The combinatorial prefactors of the diagrammatical expansion of the ϕ^4-theory have to be evaluated with the number of spin components n formally equal to 0. The ϕ^4-theory has been proven to be renormalizable to all orders of perturbation theory. The relevant parameters of a ϕ^4-theory transform back into polymer language into exactly the model (2.1). The coupling u in (2.1) is identical with the coupling u of ϕ^4-theory. The mass m of ϕ^4-theory is the conjugate quantity to the chain length N under the Laplace transformation.

Knowing this connection, one also can work directly in polymer language. For polymer statics this is a matter of convenience, while for polymer dynamics this will be the only choice, since there we know of no renormalizable dynamical field theory to which we could transform.

2.2. Some Features of Polymer Statics in Quenched Disorder

We now incorporate quenched random potentials into this standard model. Each segment tests the local potential, and the chain configurations get an additional statistical weight according to the total potential exerted on the chain. Mathematically this is done by adding to the Hamiltonian the term

$$\mathscr{H}_v[\mathbf{r}, V(\mathrm{r})] = \sum_{i=1}^{N} V(\mathrm{r}_i) \tag{2.3}$$

We assume a local Gaussian distribution of the potentials

$$\mathscr{P}_v[V] = \mathscr{N} \exp\left[-\int d^d\mathrm{r} \frac{V^2(\mathrm{r})}{2vl^d} \right] \tag{2.4}$$

and we will prove in Section 3.4 that (2.4) is the relevant contribution of any distribution of finite, short-ranged correlated potentials (of course only if the distribution is translationally invariant).

(Note in comparison that for a so-called directed polymer in quenched disorder the excluded-volume interaction term $\mathscr{H}_u[\mathbf{r}]$ is missing and the random potential is independent for each segment, $\mathscr{H}_v[\mathbf{r}, V(\mathrm{r})]_{\text{dir.pol.}} = \sum_i V(\mathrm{r}_i, i)$, $\overline{V(\mathrm{r}_i, i)\, V(\mathrm{r}_j, j)} \propto \delta_{ij}\, \delta(\mathrm{r}_i - \mathrm{r}_j)$.)

We will now first discuss the static properties of polymers with the Boltzmannian weight

$$\mathscr{P}_V[\mathbf{r}] = \mathscr{N} \exp\{ -\mathscr{H}_0[\mathbf{r}] - \mathscr{H}_u[\mathbf{r}] - \mathscr{H}_v[\mathbf{r}, V(\mathrm{r})] \} \tag{2.5}$$

since any polymer dynamics in quenched random potentials should contain this static model as a closed substructure. We summarize in the remainder

of the section our earlier discussion of quenched static expectation values in ref. 11 and especially ref. 2 (which see for further references).

One has to distinguish quenched or annealed averages. Annealed averages correspond to fluctuating potentials. The averaging over disorder then is done together with averaging over the chain configurations. This yields simple Gaussian integrals, which can be fully evaluated. As a result it is found that annealed averages are equivalent to the theory without random potentials with only the couplings replaced such as

$$u \to w := u - v \tag{2.6}$$

In other words: The theory without random potentials (2.1) has the two parameters $(u, N)_l$ on the scale l and the theory with fluctuating random potentials has the two parameters $(w, N)_l$. The shift of the coupling can be interpreted such that averaging over the potentials generates an additional attractive interaction. Therefore the chains collapse earlier, resp. the Θ temperature is higher due to the fluctuating potentials.[13] The mathematical structure above and at the collapse, however, is identical. Below the Θ-temperature for $u<0$ as well as for $w<0$ the connection to field theory breaks down and the behavior becomes nonuniversal.

Quenched averages apply to frozen-in potentials as will be dealt with in the present paper. In this case it is not the partition function that has to be averaged, but the free energy. Usually this average is difficult to treat. In the present case we observe, however, that quenched averages at zero segment density are identical with annealed averages, because the partition sum in the denominator is self-averaging. This property is specific to quenched potential averaging for geometric objects at concentration zero and differs from the behavior of field theory in random potentials, resp. with fluctuating masses, where the free energy can be self averaging, but not the partition function. Self-averaging of the partition function for lattice polymer models has long been known in the Monte Carlo literature. A rigorous proof for certain lattice models can be found in ref. 8, though there erroneously attributed to Harris.[14] The extension to general models can be found in ref. 2.

We conclude that for static expectation values of a finite number of chains of large, but finite length in an infinite embedding volume, quenched and annealed averages coincide and differ from the theory without potentials only by the shifted couplings (2.6). Especially u and v do not play any distinct role, but add up to only one coupling w. (The same is true also for all higher order couplings, resp. higher order moments of disorder,[13] which are irrelevant, however.) In dynamics, in contrast, the coupling v will play a separate role beside the static coupling w, as will be shown below.

2.3. Introduction of the Dynamic Model

We first extend the static model of Eqs. (2.1) and (2.3) by another contribution to the Hamiltonian:

$$\mathcal{H}_\xi[\mathbf{r}] = \frac{R_{\mathrm{cm}}^2[\mathbf{r}]}{2\xi^2} \tag{2.7}$$

It harmonically binds the center of mass of a polymer

$$\mathrm{R}_{\mathrm{cm}}[\mathbf{r}] = \frac{1}{N}\sum_{i=1}^{N} \mathrm{r}_i \tag{2.8}$$

to the origin. ξ^d can be interpreted as proportional to the available part of embedding space. ξ is always assumed to be much larger than any other length scale.

$\mathcal{H}_\xi[\mathbf{r}]$ in (2.7) is introduced mainly for technical purposes, since it will regularize the center-of-mass motion in Section 3.2. It is also useful for discussing the zero-segment-density limit of the last section quantitatively as well as other useful limits of the theory.[2]

Accordingly the total Hamiltonian to be considered below is

$$\mathcal{H}[\mathbf{r}, V]_\xi = \mathcal{H}_0[\mathbf{r}] + \mathcal{H}_u[\mathbf{r}] + \mathcal{H}_v[\mathbf{r}, V(\mathrm{r})] + \mathcal{H}_\xi[\mathbf{r}] \tag{2.9}$$

with Eqs. (2.1), (2.3), (2.4), and (2.7). Note that this Hamiltonian is dimensionless when r_i, l and ξ are chosen of identical dimension and u, $V(\mathrm{r})$, and v are dimensionless.

Now a Langevin dynamics

$$\frac{\partial}{\partial t} r_{i\mu}(t) = \gamma\left(-\frac{\partial \mathcal{H}[\mathbf{r}(t), V]_\xi}{\partial r_{i\mu}} + f_{i\mu}(t)\right) \tag{2.10}$$

is introduced. Equation (2.10) can be understood as an overdamped equation of motion: The displacement of a segment is proportional to the sum of a deterministic plus a thermally fluctuating force.

Had we additionally introduced a mass term with a second time derivative of $r_{i\mu}(t)$ on the left-hand side of (2.1), we would have found that by power counting arguments similar to those discussed in Section 3.4, this term is irrelevant in the RG sense on large scales, as long as the term with the first time derivative is present.

Within the approach of Eq. (2.10) it is consistent to assume the thermal forces $f_{i\mu}(t)$ to be short-range correlated in the intrinsic metric i of the chain and in time t and independent in the Cartesian components μ. The

relevant contribution of such a distribution of random forces is represented by a Gaussian ansatz

$$\mathscr{P}_f[\mathbf{f}] = \mathscr{N} \exp\left[-\frac{\gamma}{4} \int dt \sum_{i=1}^{N} \sum_{\mu=1}^{d} f_{i\mu}^2(t) \right] \tag{2.11}$$

as will be shown in Section 3.4. The variance of the distribution (2.11) is determined by the dynamical equation (2.10) and its connection to the Boltzmannian equilibrium ensemble $\exp(-\mathscr{H}[\mathbf{r}(t), V]_\xi)$, (2.9), through the Einstein relation. Then $2dy$ is the diffusion constant of an independent segment. For $\mathscr{H}[\mathbf{r}(t), V]_\xi = \mathscr{H}_0[\mathbf{r}(t)]$ the model reduces to the Rouse model of polymer dynamics.

As already discussed in the introduction, a dynamic model without hydrodynamic interaction frequently is considered as only a first step towards a more realistic dynamics, which also should take the dynamics of the solvent into account. For semidilute solutions, on the other hand, it has been argued that hydrodynamic interactions are screened by the interpenetrating segments of other chains, such that a model without hydrodynamic interactions then actually is consistent with experiments.[6] Similarly we can argue for the random potentials: If we assume that the flow of the solvent due to the displacements of the chain segments in a random potential background adds up to some random force, we are back to the model characterized by (2.10) and (2.11).

As far as the dynamics of the polymer is concerned, our model is on a similar footing as reptation theories, which also are based on a Langevin description of the segment displacements and neglect hydrodynamic interactions. Reptation approaches and our local random potential model differ in how the random medium is modeled. Reptation models focus on the topological constraints due to the random medium, which are assumed to confine the polymer to some average tube. Our model, in contrast, concentrates on the local energetic and entropic effects due to the medium as already discussed in the introduction.

Within the class of local random potential models no further approximation schemes are required, since our model can be dealt with by renormalization group methods. It represents the universal content of any local random potential theory on scales much larger than the microscopic length scale.

3. THE DYNAMIC GENERATING FUNCTIONAL

So far we have discussed the motivation for treating exactly this model. In the present section we derive the tools of the analysis.

We analyze the model by renormalization group (RG) methods, while it is actually not clear whether the model is renormalizable. The calculation therefore serves two purposes: for the verification of the renormalizability hypothesis to one-loop order, and for the derivation of quantitative results.

Having the successful extension of static ϕ^4-theory to various dynamical models in mind, this procedure will be imitated here for polymer dynamics. The method of dynamic generating functionals will be used in the form derived by Bausch *et al.*[15] It was used earlier for polymer dynamics,[5] but the precise tools as represented in this section and needed in the following ones have not been available so far.

The program of the section is in close correspondence to ref. 15: First a dynamic generating functional is derived from the Langevin equation and the distribution of thermal random forces. Then the free theory ($u=0=v$) is exactly diagonalized. It is also known as the Rouse theory of polymer dynamics. Then a perturbation theory in the couplings is set up about this free theory. Finally the continuous chain limit and the relevance of the parameters in the RG sense are discussed and a short reminder on the ε-expansion technique is included.

This establishes the tools for analyzing various correlation functions in the forthcoming sections, which can be calculated by fixing the external generating fields appropriately.

3.1. From the Langevin Equation to the Generating Functional

The notation of the Langevin equation (2.10) can be made more compact by writing $\mathbf{r}$ for the $(d\times N)$-dimensional vector of a polymer configuration (2.2), $\nabla_{\mathbf{r}}$ for the corresponding $(d\times N)$-dimensional derivative, and $\mathbf{f}$ for the random forces.

Just as in a dynamical field theory, we discretize the temporal derivative into infinitesimal time steps τ and find from (2.10)

$$\frac{\mathbf{r}(t+\tau)-\mathbf{r}(t)}{\gamma\tau} = -\alpha\,\nabla_{\mathbf{r}}\mathscr{H}[\mathbf{r}(t+\tau),\, V]_{\xi} - (1-\alpha)\,\nabla_{\mathbf{r}}\mathscr{H}[\mathbf{r}(t),\, V]_{\xi} + \mathbf{f}(t) \tag{3.1}$$

As usual we are free to choose α deliberately within the interval $[0, 1]$. The value $\alpha=1/2$ corresponds to the usual differential approach or the Stratonovich calculus, while $\alpha=0$ corresponds to the Ito calculus. Physical observables are known to be independent of the choice of α, if the calculation is done consistently with one value of α.

The Langevin equation (3.1) together with the distribution of random forces (2.11) determine the transition probability $\mathscr{P}(\mathbf{r}(t+\tau)\,|\,\mathbf{r}(t))_{V,\xi}$ that a given polymer configuration $\mathbf{r}(t)$ evolves into a configuration $\mathbf{r}(t+\tau)$ within the time τ in a given potential field $V(\mathrm{r})$ and volume ξ^d. To derive a formal expression for $\mathscr{P}$, (2.11) is first discretized in infinitesimal time steps τ and linearized by introducing the additional Gaussian variable $\tilde{\mathbf{r}}$:

$$\begin{aligned}\mathscr{P}_f[\mathbf{f}(t)]\, d^{dN}\mathbf{f}(t) &= d^{dN}\mathbf{f}(t)\,(\gamma\tau/4\pi)^{dN/2}\exp[-\gamma\tau\mathbf{f}^2(t)/4] \\ &= d^{dN}\mathbf{f}(t)\int d^{dN}\tilde{\mathbf{r}}(t)\,(\gamma\tau/2\pi)^{dN}\,\mathrm{e}^{-\gamma\tau[\tilde{\mathbf{r}}^2(t)-i\tilde{\mathbf{r}}(t)\,\mathbf{f}(t)]}\end{aligned} \tag{3.2}$$

Then $\mathbf{f}(t)$ is substituted by $\mathbf{r}(t+\tau)$ for fixed $\mathbf{r}(t)$ by means of Eq. (3.1). It is straightforward to determine the Jacobi determinant of the substitution as

$$\begin{aligned}\left|\frac{\partial f_{i\mu}(t)}{\partial r_{j\nu}(t+\tau)}\right| &= \left|\frac{\delta_{ij}\delta_{\mu\nu}}{\gamma\tau}+\alpha\frac{\partial^2\mathscr{H}[\mathbf{r}(t+\tau),V]_\xi}{\partial r_{i\mu}(t+\tau)\,\partial r_{j\nu}(t+\tau)}\right| \\ &= (\gamma\tau)^{-dN}\,[\exp\{\alpha\gamma\tau\,\nabla_{\mathbf{r}}^2\mathscr{H}[\mathbf{r}(t+\tau),V]_\xi\}+\mathscr{O}(\tau^2)]\end{aligned} \tag{3.3}$$

The choice of $\alpha=0$ reduces the determinant to a trivial factor of $(\gamma\tau)^{-dN}$. In the sequel $\alpha=0$ or the Ito calculus will be used.

The probability that a given configuration $\mathbf{r}(t)$ transforms into another configuration $\mathbf{r}(t+\tau)$ within time τ thus is given as

$$\begin{aligned}\mathscr{P}(\mathbf{r}(t+\tau)\,|\,\mathbf{r}(t))_{V,\xi}\; d^{dN}\mathbf{r}(t+\tau) &= d^{dN}\mathbf{r}(t+\tau)\int\frac{d^{dN}\tilde{\mathbf{r}}(t)}{(2\pi)^{dN}}\;\mathrm{e}^{-\gamma\tau[\tilde{\mathbf{r}}^2(t)-i\tilde{\mathbf{r}}(t)\,\dot{\mathscr{L}}(\mathbf{r},t)_{V,\xi}]}\end{aligned} \tag{3.4}$$

with

$$\dot{\mathscr{L}}(\mathbf{r},t)_{V,\xi}:=\frac{\mathbf{r}(t+\tau)-\mathbf{r}(t)}{\gamma\tau}+\nabla_{\mathbf{r}(t)}\mathscr{H}[\mathbf{r}(t),V]_\xi \tag{3.5}$$

The subscript V,ξ is used to indicate that the transition is calculated within a fixed potential configuration $V(\mathrm{r})$ and embedding volume ξ^d.

The temporal evolution within each potential configuration is Markovian. The transition probability within the finite time $T=M\tau$ therefore can be derived as

$$\mathscr{P}(\mathbf{r}(t+T)\,|\,\mathbf{r}(t))_{V,\xi}=\int\prod_{m=1}^{M-1} d^{dN}\mathbf{r}(t+m\tau)$$
$$\times\prod_{m=1}^{M}\mathscr{P}(\mathbf{r}(t+m\tau)\,|\,\mathbf{r}(t+(m-1)\,\tau))_{V,\xi} \quad (3.6)$$

or in the continuous time limit as

$$\mathscr{P}(\mathbf{r}(t+T)\,|\,\mathbf{r}(t))_{V,\xi}$$
$$=\int_{\mathbf{r}(t)}^{\mathbf{r}(t+T)} d[\mathbf{r},\tilde{\mathbf{r}}]$$
$$\times\exp\left\{-\int_t^{t+T}\gamma\,d\tau\left[\tilde{\mathbf{r}}^2(\tau)-i\tilde{\mathbf{r}}(\tau)\left(\frac{\partial\mathbf{r}(\tau)}{\gamma\,\partial\tau}+\nabla_{\mathbf{r}}\mathscr{H}[\mathbf{r}(\tau),V]_{\xi}\right)\right]\right\} \quad (3.7)$$

if expressions are understood in the $\alpha=0$ discretization. For $\alpha=0$ the normalization of the integration measure $d[\mathbf{r},\tilde{\mathbf{r}}]$ consists of powers of $(2\pi)^{-1}$ only. The probabilities (3.4)–(3.7) are then properly normalized.

One now couples external fields linearly to the fluctuating variables and averages over the fluctuations. The result is called the dynamical generating functional and is written as

$$\mathscr{Z}_1[\mathbf{h},\tilde{\mathbf{h}}]=\int d[\mathbf{r},\tilde{\mathbf{r}}]\exp\left\{-\int dt\,[\gamma\tilde{\mathbf{r}}^2-i\tilde{\mathbf{r}}(\dot{\mathbf{r}}+\gamma\nabla_{\mathbf{r}}\mathscr{H}[\mathbf{r},V]_{\xi})-i(\mathbf{hr}+\tilde{\mathbf{h}}\tilde{\mathbf{r}})]\right\}$$
$$=\left\langle\exp\left\{i\int dt\,(\mathbf{hr}+\tilde{\mathbf{h}}\tilde{\mathbf{r}})(t)\right\}\right\rangle \quad (3.8)$$

The angular brackets denote the average over the thermal forces, resp. over the variables $\mathbf{r}$ and $\tilde{\mathbf{r}}$. The lower index 1 in $\mathscr{Z}_1[\mathbf{h},\tilde{\mathbf{h}}]$ denotes a functional for one chain. We later will also deal with a functional for two chains. The external fields $\mathbf{h}$ and $\tilde{\mathbf{h}}$ can be chosen freely according to the problem. Three different choices will generate three different correlation functions, which will be discussed in the next sections.

We now set up the general framework of perturbation theory. This first of all requires us to analyze the Langevin kernel

$$\left(\frac{\partial\mathbf{r}(t)}{\gamma\,\partial t}+\nabla_{\mathbf{r}}\mathscr{H}[\mathbf{r}(t),V]_{\xi}\right)$$

We find

$$\frac{\partial\mathscr{H}_0[\mathbf{r}]}{\partial r_{i\mu}}=\frac{2r_{i\mu}-r_{i-1\mu}-r_{i+1\mu}}{2l^2} \quad (3.9)$$

and

$$\frac{\partial \mathscr{H}_\xi[\mathbf{r}]}{\partial r_{i\mu}} = \frac{R_{\mathrm{cm},\mu}}{N\xi^2} = \frac{1}{N^2\xi^2} \sum_{j=1}^{N} r_{j\mu} \tag{3.10}$$

if we formally use

$$\mathrm{r}_0 = \mathrm{r}_1 \qquad \text{and} \qquad \mathrm{r}_{N+1} = \mathrm{r}_N \tag{3.11}$$

The contributions (3.9), (3.10), and $\partial_t \mathbf{r}$ are linear in r and can be treated exactly. They constitute the free diagonalizable theory. This theory up to the finite-size term (3.10) is the so-called Rouse theory of polymer dynamics.

The other two terms of the Hamiltonian will be treated perturbatively. The excluded-volume interaction leads to

$$\begin{aligned} \frac{\partial \mathscr{H}_u[\mathbf{r}]}{\partial r_{i\mu}} &= \frac{\partial}{\partial r_{i\mu}} u l^d \sum_{1 \leqslant j < k \leqslant N} \int \frac{d^d\mathrm{p}}{(2\pi)^d} \, \mathrm{e}^{i\mathrm{p}(\mathrm{r}_j - \mathrm{r}_k)} \\ &= u l^d \sum_{k=1, k \neq i}^{N} \int_{\mathrm{p}} i p_\mu \mathrm{e}^{i\mathrm{p}(\mathrm{r}_i - \mathrm{r}_k)} \end{aligned} \tag{3.12}$$

The random potential contribution is

$$\frac{\partial \mathscr{H}_v[\mathbf{r}, V]}{\partial r_{i\mu}} = \frac{\partial}{\partial r_{i\mu}} \sum_{j=1}^{N} \int_{\mathrm{p}} \mathrm{e}^{-i\mathrm{p}\mathrm{r}_j} V^{\mathrm{F}}(\mathrm{p}) = -\int_{\mathrm{p}} i p_\mu \, \mathrm{e}^{-i\mathrm{p}\mathrm{r}_i} V^{\mathrm{F}}(p) \tag{3.13}$$

with the Fourier transformation

$$V^{\mathrm{F}}(\mathrm{p}) := \int d^d\mathrm{r} \, \mathrm{e}^{i\mathrm{p}\mathrm{r}} V(\mathrm{r}), \qquad V(\mathrm{r}) = \int_{\mathrm{p}} \mathrm{e}^{-i\mathrm{p}\mathrm{r}} V^{\mathrm{F}}(\mathrm{p}), \qquad \int_{\mathrm{p}} := \int \frac{d^d\mathrm{p}}{(2\pi)^d} \tag{3.14}$$

The complete V-dependent expression in the exponent of (3.8) is

$$\begin{aligned} &-\int dt \, (-i\tilde{\mathbf{r}} \, \gamma \, \nabla_{\mathrm{r}} \mathscr{H}_v[\mathbf{r}, V]) \\ &= \gamma \int dt \sum_{i=1}^{N} \int_{\mathrm{p}} (\mathrm{p}\tilde{\mathrm{r}}_i(t)) \, \mathrm{e}^{-i\mathrm{p}\mathrm{r}_i(t)} V^{\mathrm{F}}(\mathrm{p}) =: -\int_{\mathrm{p}} V^{\mathrm{F}}(\mathrm{p}) \, C^{\mathrm{F}}(-\mathrm{p}) \end{aligned} \tag{3.15}$$

with

$$C^{\mathrm{F}}(-\mathrm{p}) = -\int \gamma \, dt \sum_{i=1}^{N} (\mathrm{p}\tilde{\mathrm{r}}_i(t)) \, \mathrm{e}^{-i\mathrm{p}\mathrm{r}_i(t)} = -C^{\mathrm{F}*}(\mathrm{p}) \tag{3.16}$$

Averaging $\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]$ with the potential distribution (2.4), we find

$$\begin{aligned}
&\int d[V]\, \mathscr{P}_v[V] \exp\left\{-\int_{\mathrm{p}} V^{\mathrm{F}}(\mathrm{p})\, C^{\mathrm{F}}(-\mathrm{p})\right\} \\
&\quad = \exp\left\{\tfrac{1}{2} v l^d \int_{\mathrm{p}} C^{\mathrm{F}}(\mathrm{p})\, C^{\mathrm{F}}(-\mathrm{p})\right\} \\
&\quad = \exp\left\{-\tfrac{1}{2} v l^d \int_{\mathrm{p}} |C^{\mathrm{F}}(\mathrm{p})|^2\right\} \qquad (3.17)
\end{aligned}$$

Obviously, quenched averaging can be performed here without problems, as is characteristic for dynamical functionals.

Note that (3.17) is bounded from above for any positive second moment v, i.e., in the physical parameter regime. This is in contrast to the disorder average of the static weight (2.5), which is not bounded for $w = u - v < 0$.

The potential averaged generating functional now can be written as

$$\overline{\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]} = \int d[\mathbf{r}, \tilde{\mathbf{r}}] \exp\left\{-S_0 - \hat{S}_I^{(1)} + i \int dt\, (\mathbf{h}\mathbf{r} + \tilde{\mathbf{h}}\tilde{\mathbf{r}})\right\} \qquad (3.18)$$

The free action is

$$S_0 = \int dt \sum_{i=1}^{N} \left(\gamma \tilde{\mathrm{r}}_i^2 - i\tilde{\mathrm{r}}_i \left(\dot{\mathrm{r}}_i + \gamma \frac{2\mathrm{r}_i - \mathrm{r}_{i-1} - \mathrm{r}_{i+1}}{2l^2} + \gamma \frac{\mathrm{R}_{\mathrm{cm}}[\mathbf{r}]}{N\xi^2}\right)\right)(t) \qquad (3.19)$$

and the interactions take the form

$$\begin{aligned}
\hat{S}_I^{(1)} = &\int_{\mathrm{p}} u l^d \int \gamma\, dt \sum_{i,j=1, j\neq i}^{N} (\mathrm{p}\tilde{\mathrm{r}}_i(t))\, \mathrm{e}^{i\mathrm{p}(\mathrm{r}_i - \mathrm{r}_j)(t)} \\
&+ \tfrac{1}{2} \int_{\mathrm{p}} v l^d \int \gamma^2\, dt\, dt' \sum_{i,j=1}^{N} (\mathrm{p}\tilde{\mathrm{r}}_i(t))\, (\mathrm{p}\tilde{\mathrm{r}}_j(t'))\, \mathrm{e}^{i\mathrm{p}(\mathrm{r}_i(t) - \mathrm{r}_j(t'))} \qquad (3.20)
\end{aligned}$$

In the last formula the very different character of the two interactions should be noted. This is again in contrast to polymer statics: In Section 2.2 we recalled that polymer statics in the appropriate limit has only one coupling $w = u - v$, while here the two couplings appear with a clearly distinguished temporal dependence. This can be understood on physical grounds: Segments avoid each other when they are at the same place at the same time.

A time-independent local potential, however, becomes effective whenever a segment comes to the site of the potential. Averaging over the potentials thus generates an effective segment–segment interaction, which is local in space, but nonlocal in time.

3.2. Diagonalizing the Free Theory

The free part S_0 of the generating functional (3.18) can be diagonalized by introducing the standard Rouse coordinates. We define the orthogonal transformation

$$s_{k\mu}(t) = \sum_{i=1}^{N} O_{ki}\, r_{i\mu}(t), \qquad \tilde{s}_{k\mu}(t) = \sum_{i=1}^{N} O_{ki}\, \tilde{r}_{i\mu}(t) \tag{3.21}$$

with the transformation matrix

$$O_{ki} = \begin{cases} \left(\frac{1}{N}\right)^{1/2} & \text{for} \quad k = 0 \\ \left(\frac{2}{N}\right)^{1/2} \cos\left(\pi k \frac{2i-1}{2N}\right) & \text{for} \quad k = 1,\ldots, N-1 \end{cases} \tag{3.22}$$

$$\sum_{k=0}^{N-1} O_{ki} O_{kj} = \delta_{ij}, \qquad \sum_{i=1}^{N} O_{ki} O_{li} = \delta_{kl} \tag{3.23}$$

Thus

$$r_{i\mu}(t) = \sum_{k=0}^{N-1} O_{ki}\, s_{k\mu}(t), \quad \text{etc.}$$

Obviously the eigenmodes s_k emerge from a Fourier analysis with respect to the intrinsic metric i and with the boundary condition (3.11). The $k = 0$ mode is the center-of-mass mode with

$$\mathrm{s}_0(t) = \sum_{i=1}^{N} \frac{1}{\sqrt{N}}\, \mathrm{r}_i(t) = \sqrt{N}\, \mathrm{R}_{\mathrm{cm}}[\mathbf{r}(t)] \tag{3.24}$$

If we furthermore also transform the external fields

$$H_{k\mu}(t) = \sum_{i=1}^{N} O_{ki}\, h_{i\mu}(t), \qquad \tilde{H}_{k\mu}(t) = \sum_{i=1}^{N} O_{ki}\, \tilde{h}_{i\mu}(t) \tag{3.25}$$

the generating functional of the free theory transforms into

$$\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]_0 = \int d[\mathbf{s}(t), \tilde{\mathbf{s}}(t)] \times \exp\left\{-\int dt \sum_{k=0}^{N-1} \sum_{\mu=1}^{d} [\gamma \tilde{s}_{k\mu}^2 - i\tilde{s}_{k\mu}(\partial_t + \omega_k) s_{k\mu}](t)\right\} \times \exp\left\{i \int dt \sum_{k=0}^{N-1} \sum_{\mu=1}^{d} [H_{k\mu} s_{k\mu} + \tilde{H}_{k\mu} \tilde{s}_{k\mu}](t)\right\} \tag{3.26}$$

with the energies

$$\omega_0 = \frac{\gamma}{N\xi^2} \tag{3.27}$$

$$\omega_k = \frac{2\gamma}{l^2} \sin^2 \frac{\pi k}{2N} \qquad \text{for} \quad k = 1, \ldots, N \tag{3.28}$$

After a Fourier transformation in time $[f(t) = \int d\omega/2\pi \exp(-i\omega t) f^{\mathrm{F}}(\omega)]$, the generating functional (3.26) is diagonal in k, μ, and ω. Its exponential is quadratic in $\tilde{s}_{k\mu}^F(\omega)$. Thus the Gaussian integration over $\tilde{s}_{k\mu}^F(\omega)$ can be performed. A quadratic exponential in $s_{k\mu}^F(\omega)$ results. Now also $s_{k\mu}^F(\omega)$ can be integrated out. The result is

$$\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]_0 = \exp\left\{-\int \frac{d\omega}{2\pi} \sum_{k=0}^{N-1} \sum_{\mu=1}^{d} \left(\frac{\gamma H_{k\mu}^{\mathrm{F}}(\omega) H_{k\mu}^{\mathrm{F}}(-\omega)}{\omega^2 + \omega_k^2} - \frac{\tilde{H}_{k\mu}^{\mathrm{F}}(\omega) H_{k\mu}^{\mathrm{F}}(-\omega)}{\omega + i\omega_k}\right)\right\} \tag{3.29}$$

(3.29) can be Fourier transformed back to t-variables

$$\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]_0 = \exp\left(-\tfrac{1}{2} \int dt\, dt' \sum_{k=0}^{N-1} \sum_{\mu=1}^{d} H_{k\mu}(t)\, G_k(t-t')\, H_{k\mu}(t')\right) \times \exp\left(-\int dt\, dt' \sum_{k=0}^{N-1} \sum_{\mu=1}^{d} H_{k\mu}(t)\, R_k(t-t')\, \tilde{H}_{k\mu}(t')\right) \tag{3.30}$$

We introduce the Green's function

$$G_k(t) = \gamma \int \frac{d\omega}{\pi} \frac{1}{\omega^2 + \omega_k^2} \, \mathrm{e}^{-i\omega t} = \frac{\gamma}{\omega_k} \, \mathrm{e}^{-\omega_k |t|} \tag{3.31}$$

and the response function

$$R_k(t) = -\int \frac{d\omega}{2\pi} \frac{1}{\omega + i\omega_k} \, \mathrm{e}^{-i\omega t} = i\theta(t) \, \mathrm{e}^{-\omega_k t} \tag{3.32}$$

Obviously

$$R_k(t) = \theta(t) \frac{\partial G_k(t)}{i\gamma \, \partial t} \tag{3.33}$$

holds.

The θ-function is required to appear in (3.32) for fulfilling causality.[15, 16] Note, that causality here automatically holds, since the pole $\omega + i\omega_k$ lies in the lower half of the analytic plane. The convergence requirement leading to the closure of the path of integration in the upper or lower half of the analytic plane can be traced back to the Langevin kernel $(\partial_t + \omega_k)$ converging only for $t > 0$ as long as $\omega_k > 0$. It is a direct consequence of the original Langevin equation, which relaxes in the positive time direction.

Note further that we used $\omega_k > 0$. This would not hold for the energy ω_0, (3.27), of the center-of-mass mode in the limit of infinite volume $\xi^d \to \infty$. We have circumvented the problem by regularizing the $k = 0$ mode via the harmonic potential (2.7).

The connection of the Green's and response functions to expectation values can be established as usual by the observation that $G_k(t - t')$ can be derived from (3.30) by

$$G_k(t - t') = \frac{\delta^2}{i \, \delta H_{k\mu}(t) \, i \, \delta H_{k\mu}(t')} \bigg|_{\mathbf{H} = \tilde{\mathbf{H}} = 0} \mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]_0 \tag{3.34}$$

Applying these derivatives to the representation (3.26) of $\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]_0$, we find

$$\langle s_{k\mu}(t) \, s_{k'\mu'}(t') \rangle_0 = \delta_{kk'} \delta_{\mu\mu'} G_k(t - t') \tag{3.35}$$

and similarily

$$\langle s_{k\mu}(t) \, \tilde{s}_{k'\mu'}(t') \rangle_0 = \delta_{kk'} \delta_{\mu\mu'} R_k(t - t') \tag{3.36}$$

For symmetry reasons [cf. (3.8) and (3.26)], the following relation holds:

$$\langle s_{k\mu}(t) \rangle_0 = 0 = \langle \tilde{s}_{k\mu}(t) \rangle_0 \tag{3.37}$$

The subscript 0 on the angular brackets as well as on the generating functional refers to the free theory.

Now the value of $\Theta(0)$ in (3.32) is still undefined. Inserting the identity (3.36) into the time-discretized equations, we easily derive

$$\Theta(0) = \alpha = 0 \tag{3.38}$$

The generating functional of the free theory can finally be written in terms of the segment coordinates by reversing the transformation (3.21)–(3.25):

$$\mathscr{Z}_1[\mathbf{h}, \tilde{\mathbf{h}}]_0 = \exp\left(-\tfrac{1}{2}\int dt\, dt' \sum_{i,j=1}^{N} \sum_{\mu=1}^{d} h_{i\mu}(t)\, G_{ij}(t-t')\, h_{j\mu}(t')\right)$$
$$\times \exp\left(-\int dt\, dt' \sum_{i,j=1}^{N} \sum_{\mu=1}^{d} h_{i\mu}(t)\, R_{ij}(t-t')\, \tilde{h}_{j\mu}(t')\right) \tag{3.39}$$

with

$$G_{ij}(t) = \sum_{k=0}^{N-1} O_{ki} O_{kj} G_k(t) = \langle r_{i\mu}(t+t_0)\, r_{j\mu}(t_0)\rangle_0 \tag{3.40}$$

$$R_{ij}(t) = \sum_{k=0}^{N-1} O_{ki} O_{kj} R_k(t) = \theta(t) \frac{\partial G_{ij}(t)}{i\gamma\, \partial t} \tag{3.41}$$

The eigenmode representation of G and R is denoted by one and the segment representation by two subscripts.

3.3. Perturbation Expansion in the Couplings

Now the remaining part of the action (3.20) due to the interactions u and v needs to be rewritten. First the exponential function $\exp(-\hat{S}_I^{(1)})$ in (3.18) is Taylor expanded. Then it is a standard procedure to replace $\tilde{r}_{i\mu}(t)$ by the functional derivative $\delta/i\delta\tilde{h}_{i\mu}(t)$. The equivalence can be read directly from (3.18). The treatment of $r_{i\mu}(t)$ is nonstandard, because it still is exponentiated after the Taylor expansion of $\exp(-\hat{S}_I^{(1)})$. Now the observation is helpful that the coefficient p_μ of $r_{i\mu}(t)$ appears in (3.20) in the very same way as an external field $h_{i\mu}(t)$. It therefore can be taken into account by simply adding additional internal fields, which are integrated over. For distinction from these additional internal fields, the external fields, which determine the physical properties of the generating functional, will be denoted by $\mathbf{h}_{(0)}$ or $\tilde{\mathbf{h}}_{(0)}$ below. The perturbation theory is then expressed as

$$\overline{\mathscr{Z}_1[\mathbf{h}_{(0)}, \tilde{\mathbf{h}}_{(0)}]} = \sum_{m=0}^{\infty} \frac{1}{m!} \prod_{\sigma=1}^{m} (-S_{I(\sigma)}^{(1)})$$
$$\times \exp\left(-\frac{1}{2}\int d\tau\, d\tau' \sum_{i,j=1}^{N} \sum_{\mu=1}^{d} h_{i\mu}(\tau)_{(m)}\, G_{ij}(\tau-\tau')\, h_{j\mu}(\tau')_{(m)}\right)$$
$$\times \exp\left(-\int d\tau\, d\tau' \sum_{i,j=1}^{N} \sum_{\mu=1}^{d} h_{i\mu}(\tau)_{(m)}\, R_{ij}(\tau-\tau')\, \tilde{h}_{j\mu}(\tau')_{(0)}\right) \tag{3.42}$$

with

$$S^{(1)}_{I(\sigma)} = \frac{1}{2}\int_{\mathrm{p}_\sigma} l^d \sum_{i_\sigma, j_\sigma = 1}^{N} \int \gamma^2 \, dt_\sigma \, dt'_\sigma$$
$$\times \left\{ u \frac{\delta(t_\sigma - t'_\sigma)}{\gamma} [\mathscr{R}(i_\sigma, \mathrm{p}_\sigma, t_\sigma) + \mathscr{R}(j_\sigma, -\mathrm{p}_\sigma, t'_\sigma)] \right.$$
$$\left. - v \, \mathscr{R}(i_\sigma, \mathrm{p}_\sigma, t_\sigma) \, \mathscr{R}(j_\sigma, -\mathrm{p}_\sigma, t'_\sigma) \right\} \tag{3.43}$$

and

$$\mathscr{R}(i, \mathrm{p}, t) = \sum_{\mu=1}^{d} p_\mu \frac{\delta}{i \, \delta \tilde{h}_{i\mu}(t)_{(0)}}$$
$$= \int d\tau \sum_{j=1}^{N} \sum_{\mu=1}^{d} (p_\mu h_{j\mu}(\tau)_{(m)}) \, iR_{ji}(\tau - t) = -\mathscr{R}(i, -\mathrm{p}, t) \tag{3.44}$$

and the external and internal fields

$$h_{i\mu}(\tau)_{(m)} = h_{i\mu}(\tau)_{(0)} + \sum_{\sigma=1}^{m} p_{\sigma\mu} [\delta_{ii_\sigma} \delta(\tau - t_\sigma) - \delta_{ij_\sigma} \delta(\tau - t'_\sigma)] \tag{3.45}$$

In (3.43) additionally the segment summation in the u term has been split into a symmetric sum of two $\mathscr{R}$'s. In this way the constraint $i \neq j$ in (3.20) can be dropped, as can be verified by inserting the last identity of (3.44).

The extension of this aparatus to problems with more than one chain is straightforward and summarized in Section 5 in Eqs. (5.2)–(5.5).

3.4. The Continuous Chain Limit, the Relevance of Parameters, and the ϵ-Expansion

The continuous chain limit is the limit of vanishing segment length l. It represents one specific microscopic model, which should represent physics on large length scales just as well as the discrete model discussed so far, if universality holds. However, it simplifies the analysis in two ways. On one hand we will use a dimensional regularization scheme rather than a cutoff regularization in the RG analysis. An ε-expansion is obscured by additionally keeping a microscopic length scale in the calculation. On the other hand in the present section we want to analyze the relevance or irrelevance of the parameters of the theory at the Gaussian fixed point of RG flow. This is done by analyzing their canonical dimensions. Such a dimensional analysis most naturally can be performed in the continuous chain limit.

From the analysis of the Hamiltonian (2.9) one can derive that the limit of $l \to 0$ requires the introduction of the following continuous chain parameters:

$$S = Nl^2, \qquad s = il^2$$

$$\hat{\mathrm{r}}(s, t) = \mathrm{r}_i(t), \qquad \int_0^S \frac{ds}{l^2} \cdots + \mathcal{O}\left(\frac{l^2}{S}\right) = \sum_{i=1}^{N} \cdots \tag{3.46}$$

$$\hat{u} = 2(4\pi)^{-d/2} u l^{d-4} \tag{3.47}$$

$$\hat{V}(\mathrm{r}) = V(\mathrm{r})\, l^{-2}, \qquad \hat{v} = 2(4\pi)^{-d/2} v l^{d-4} \tag{3.48}$$

The numerical factors $2(4\pi)^{-d/2}$ are introduced for later convenience. Inserting the Hamiltonian into the dynamical functional (3.18), one furthermore has to replace

$$\hat{\tilde{\mathrm{r}}}(s, t) = \tilde{\mathrm{r}}_i(t)\, l^{-2}, \qquad \hat{\gamma} = \gamma l^2 \tag{3.49}$$

$$\hat{\mathrm{h}}(s, t) = \frac{\mathrm{h}_i(t)}{\hat{\gamma} l^2}, \qquad \hat{\tilde{\mathrm{h}}}(s, t) = \frac{\tilde{\mathrm{h}}_i(t)}{\hat{\gamma}} \tag{3.50}$$

Rewriting the action in the continuous chain variables, the l dependence vanishes completely except for the correction term $\mathcal{O}(l^2/S) = \mathcal{O}(1/N)$ due to replacing the sum over the segments by an integral. This correction can be neglected for $N \gg 1$. We find

$$\begin{aligned} -S_0 - \tilde{S}_I^{(1)} = -\int \hat{\gamma}\, dt \int_0^S ds \Bigg[\hat{\tilde{\mathrm{r}}}(s)^2 - i\hat{\tilde{\mathrm{r}}}(s) \Bigg(& \frac{\partial \hat{\mathrm{r}}(s)}{\partial \hat{\gamma} t} - \frac{\partial^2 \hat{\mathrm{r}}(s)}{2\, \partial s^2} + \frac{\mathrm{R}_{\mathrm{cm}}[\hat{\mathrm{r}}]}{S\xi^2} \\ & + \frac{\hat{u}(4\pi)^{d/2}}{2} \int_0^S ds' \int_{\mathrm{p}} ip\, \mathrm{e}^{ip(\hat{\mathrm{r}}(s) - \hat{\mathrm{r}}(s'))} - \int d^d\mathrm{r}\, \hat{V}(\mathrm{r}) \int_{\mathrm{p}} ip\, \mathrm{e}^{ip(\hat{\mathrm{r}} - \hat{\mathrm{r}}(s))} \Bigg) \\ & - i(\hat{\mathrm{h}}(s)\, \hat{\mathrm{r}}(s) + \hat{\tilde{\mathrm{h}}}(s)\, \hat{\tilde{\mathrm{r}}}(s)) \Bigg] (t) \end{aligned} \tag{3.51}$$

Note, however, that in this limit we will encounter the RG specific singularities in evaluating perturbation theory.

We now discuss the relevance of parameters by analyzing their dimensions in the unrenormalized theory, i.e., at the trivial fixed point in the RG sense. The continuous chain variables in (3.51) are dimensional quantities. One immediately finds, that $(\hat{\gamma} t)^{1/4}$, $s^{1/2}$, $S^{1/2}$, and ξ all have the dimension of a length. The system size ξ will always be assumed to be much larger than the chain size $\sqrt{S}$,

$$\xi^2 \gg S \tag{3.52}$$

The diffusion length $(\hat{\gamma}t)^{1/4}$ requires that we choose a special value of t, which can only be fixed by the external fields. It is therefore a natural choice to measure all length scales on the scale of $\sqrt{S}$ as the intrinsic parameter of the model. We then find the following dimensionless combinations:

$$\begin{aligned} 1 &= [\hat{\gamma}t\, S^{-2}] = [\xi\, S^{-1/2}] = [\hat{\mathrm{r}}(s,t)\, S^{3/2}] \\ &= [\hat{\mathrm{f}}(s,t)\, S^{-1/2}] = [\mathrm{p}\, S^{1/2}] \\ &= [\hat{u}\, S^{\varepsilon/2}] = [\hat{V}(\mathrm{r})\, S] = [\hat{\mathrm{h}}(s,t)\, S^{7/2}] = [\hat{\tilde{\mathrm{h}}}(s,t)\, S^{3/2}] \end{aligned} \tag{3.53}$$

where $\varepsilon = 4 - d$. Analyzing the relevance or irrelevance of parameters at the Gaussian fixed point is now straight forward. A perturbation theory in the excluded-volume coupling $\hat{u}$ necessarily evolves in the dimensionless parameter $\hat{u}S^{\varepsilon/2} = 2(4\pi)^{-d/2}\, uN^{2-d/2}$. For $4-d=\varepsilon>0$ this perturbation obviously is relevant, since the parameter $\hat{u}S^{\varepsilon/2}$ increases with chain length S. A similar analysis yields, that an n-segment interaction would evolve in powers of $\hat{u}_n S^{n-(n-1)d/2}$. For $d \geqslant 3$ only the 2-segment interaction $u_2 = \hat{u}$ is relevant.

We now extend our analysis to the two distribution functions $\mathscr{P}_v[V]$, (2.4), of the quenched random potentials and $\mathscr{P}_f[\mathbf{f}]$, (2.11), for the random thermal forces. We have claimed that for short-range correlated potentials or forces the Gaussian ansatze (2.4) and (2.11) represent the only relevant contributions and will prove this now.

First we treat $\mathscr{P}_v[V]$. The part of the action to be averaged over $\hat{V}(r)$ reads

$$\int d^d\mathrm{r}\, \hat{V}(\mathrm{r})\, \hat{C}(\mathrm{r})$$

with

$$\hat{C}(\mathrm{r}) = -\int \hat{\gamma}\, dt \int_0^S ds \int_{\mathrm{p}} (\mathrm{p}\hat{\tilde{\mathrm{r}}}(s,t))\, \mathrm{e}^{i\mathrm{p}(\mathrm{r}-\hat{\mathrm{r}}(s,t))} \tag{3.54}$$

We conclude that

$$[\hat{C}(\mathrm{r})\, S^{-1+d/2}] = 1 \tag{3.55}$$

Averaging over arbitrary local distributions of $\hat{V}(\mathrm{r})$ generates terms like

$$\hat{v}_n \int d^d\mathrm{r}\, \hat{C}^n(\mathrm{r})$$

for arbitrary n. Inserting (3.55), $\hat{v}_n S^{n-(n-1)d/2}$ immediately is found to be dimensionless, i.e., the moment $\hat{v}_n$ has the same dimension as the coupling $\hat{u}_n$ for given n. This also could have been concluded in a shortcut from the observation, that $\hat{C}(\mathrm{r})$ has the same canonical dimension as $\phi^2(\mathrm{r})$ in a static ϕ^4 theory or as the segment density $\rho(\mathrm{r}, t) = \int_0^S ds\, \delta^d(\mathrm{r} - \hat{\mathrm{r}}(s, t))$. Accordingly, in $d \geqslant 3$ only $\hat{v}_1$ and $\hat{v}_2$ are relevant. $\hat{v}_1$ as the first moment of the potential represents a global shift of the energy scale, which in expectation values calculated in our present canonical scheme simply cancels. $\hat{v} := \hat{v}_2$ is the relevant disorder coupling to be kept in the sequel.

Now we analyze $\mathscr{P}_f[\mathbf{f}]$. For short-range correlated forces one can think of corrections in powers of $\mathrm{f}_i^2(t)$. A possible ansatz could be

$$\mathscr{P}_{\alpha,\beta}[\mathbf{f}] = \exp\left\{ -\int \gamma\, dt \left[\tfrac{1}{4} \sum_i \mathrm{f}_i^2(t) + \alpha \sum_i (\mathrm{f}_i^2(t))^2 + \beta \sum_{i,j} \mathrm{f}_i^2(t)\, \mathrm{f}_j^2(t) + \cdots \right] \right\} \tag{3.56}$$

In the continuous chain limit the parameters and variables have to be replaced by

$$\hat{\mathrm{f}}(s, t) = \frac{\mathrm{f}_i(t)}{l^2}, \qquad \hat{\alpha} = \alpha l^4, \qquad \hat{\beta} = \beta l^2 \tag{3.57}$$

The dimensional analysis then yields

$$1 = [\hat{\alpha}\, S^{-3}] = [\hat{\beta}\, S^{-2}] \tag{3.58}$$

The corrections α and β and further ones of higher powers of $\mathrm{f}_i^2(t)$ are thus irrelevant.

We conclude, that the model introduced in Section 2 contains all parameters which are relevant in the RG sense at the Gaussian fixed point at dimension $d \geqslant 3$. The model therefore can be characterized by the four relevant parameters $(S, \hat{\gamma}, \hat{u}, \hat{v})$ for $\xi \to \infty$. In other words: These four parameters span the critical manifold in which universal laws will be derived.

We finish the section with a short reminder on the ε-expansion technique which will be used in the sequel. As outlined in the introduction and in Section 2.1, integration over the microscopic degrees of freedom up to some arbitrary scale l_R yields effective parameters on this scale, the renormalized parameters. Renormalizability implies that the microstructure up to l_R can be completely absorbed in the renormalized parameters, and the theory can then be completely rephrased in terms of the renormalized parameters on the arbitrary scale l_R in an identical functional structure. RG thus establishes the connection among equivalent theories defined on different scales.

In the continuous chain limit the microscopic scale of the bare theory *a priori* is lost, but the information needed for a construction of the RG is saved in the divergencies of the theory as the dimension tends to the upper critical dimension. They take the form of poles in $\varepsilon = 4 - d$, which have to be absorbed into a multiplicative redefinition of the parameters of the theory: Since to lowest order the renormalized parameters are proportional to the bare parameters, they can be written as proportional to the bare parameters times Z factors, which absorb the contribution of the perturbation expansion of the integrated microscopics. For polymer statics one writes more precisely

$$\bar{u}(l_R)\, l_R^{-\varepsilon} = \hat{u}\, Z_u^{-1}(\bar{u}(l_R)), \qquad S_R(l_R) = S\, Z_s^{-1}(\bar{u}(l_R)) \tag{3.59}$$

[The perturbation expansion requires a dimensionless coupling constant. u was introduced as dimensionless in Section 2.1. Thus $\hat{u}$ in (3.47) has dimension $S^{-\varepsilon/2}$ and $\bar{u}(l_R)$ is the renormalized dimensionless coupling constant on the scale l_R.] The Z factors are Taylor expandable in the couplings of the theory. The coefficients of the expansion depend on ε. We will use the minimal subtraction scheme in absorbing only the principal part of a Laurent expansion in ε in the Z factors. Then, e.g., $Z_s(\bar{u}(l_R))$ reads

$$Z_s(\bar{u}(l_R)) = 1 - \frac{\bar{u}(l_R)}{\varepsilon} + \mathcal{O}(\bar{u}^2) \tag{3.60}$$

4. THE CENTER-OF-MASS MOTION

With the mathematical tools derived in the last section, we now will calculate physical expectation values to first nontrivial order of perturbation theory (one loop). In this section we consider the center-of-mass motion and in Section 6 the internal modes. Besides these quantities, expectation values of two or more polymers are also of physical as well as mathematical interest. Such a quantity will be discussed in Section 5. These calculations also are best for the RG analysis.

4.1. The Tree Approximation

Consider the external field

$$h_{i\mu}(\tau)_{\mathrm{cm}} = -\frac{q_\mu}{N}(\delta(t-\tau) - \delta(t'-\tau)) \tag{4.1}$$

Inserting it into (3.8), we find

$$\overline{\mathscr{Z}_1[\mathbf{h}_{cm}, \tilde{\mathbf{h}} \equiv 0]} = \overline{\langle e^{-iq(R_{cm}[\mathbf{r}(t)] - R_{cm}[\mathbf{r}(t')])} \rangle}$$

$$= \int d^d r \, e^{-iqr} \overline{\langle \delta^d(R_{cm}[\mathbf{r}(t)] - R_{cm}[\mathbf{r}(t')] - r) \rangle}$$

$$=: \overline{\mathscr{G}_{cm}(q, t-t')} \tag{4.2}$$

It yields all moments of the distance the center of mass of a polymer diffuses within time $t-t'$, since it is the Fourier-transformed distance distribution function.

The tree approximation, i.e., the contribution of the free theory, is found by inserting the external field into (3.39). If we further use (3.27), (3.31), (3.40), and the property

$$\sum_{i=1}^{N} O_{ki} = \sqrt{N} \sum_{i=1}^{N} O_{0i} O_{ki} = \sqrt{N} \, \delta_{k0} \tag{4.3}$$

of the orthogonal transformation matrix, we derive

$$\mathscr{Z}_1[\mathbf{h}_{cm}, \tilde{\mathbf{h}} \equiv 0]_0 = \exp(-q^2 \xi^2 \{1 - \exp[-\gamma \, |t-t'|/(N\xi^2)]\}) \tag{4.4}$$

On the other hand, by means of (3.24)–(3.26), (3.35), and (4.3) we find

$$\mathscr{Z}_1[\mathbf{h}_{cm}, \tilde{\mathbf{h}} \equiv 0]_0 = \exp\left\{ -\frac{q^2}{2d} \langle (R_{cm}[\mathbf{r}(t)] - R_{cm}[\mathbf{r}(t')])^2 \rangle_0 \right\} \tag{4.5}$$

The subscript 0 always refers to the free theory.

In general the second moment can be derived from (4.2) as

$$R^2(t) := \overline{\langle (R_{cm}[\mathbf{r}(t+t_0)] - R_{cm}[\mathbf{r}(t_0)])^2 \rangle} = -\Delta_q |_{q=0} \, \mathscr{G}_{cm}(q, t) \tag{4.6}$$

In tree approximation this second moment is obviously

$$R^2(t)_0 = 2d \, \xi^2 (1 - e^{-\Gamma |t|/\xi^2}) \tag{4.7}$$

Here we introduce the notation

$$\Gamma = \frac{\gamma}{N} = \frac{\hat{\gamma}}{S} \tag{4.8}$$

γ/N refers to the discrete chain and $\hat{\gamma}/S$ to the continuous chain limit (3.46), (3.49).

In (4.4), resp. (4.7), two limits can be distinguished:

(i) For $\Gamma|t| \ll \xi^2$, $R^2(t)_0$ reads

$$R^2(t)_0 = 2d\,\Gamma|t| \left\{1 + \mathcal{O}\left(\frac{\Gamma|t|}{\xi^2}\right)\right\} \tag{4.9}$$

$2d\Gamma|t|$ therefore can be identified with the free mean square diffusion length of the center of mass within time t. The corrections to free diffusion due to the finiteness of the available embedding space ξ^d are not yet important.

(ii) For $\Gamma|t| \gg \xi^2$, on the other hand, the temporal correlations decay and the center-of-mass positions equilibrate independently in space ξ^d, yielding

$$R^2(t)_0 = 2d\,\xi^2\{1 - \mathcal{O}(e^{-\Gamma|t|/\xi^2})\} \tag{4.10}$$

The last equation allows us to discuss the question of whether the calculation is performed in thermal equilibrium. If no initial condition is specified at any finite time, the calculation implies any initial condition at time $t = -\infty$. At any finite time the system is thus in thermal equilibrium if all relaxation times are finite. But the largest relaxation time is that of the center of mass [cf. (3.27) and (3.28)]. For $\Gamma|t| \gg \xi^2$, therefore, equilibrium is reached generally. We call this limit the ergodic limit.[2] (Strictly speaking this property should even be called mixing.)

4.2. Perturbation Theory

Corrections to the trivial result (4.7) necessarily involve the disorder. Indeed, the equation of motion of the center of mass can be derived directly from the Langevin equation (2.10) as

$$\frac{\partial}{\partial t} R_{\mathrm{cm},\mu} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial t} r_{i\mu} = \Gamma\left(-\frac{R_{\mathrm{cm},\mu}}{\xi^2} - \sum_{i=1}^{N} \frac{\partial V(\mathbf{r}_i)}{\partial r_{i\mu}} + \sum_{i=1}^{N} f_{i\mu}\right) \tag{4.11}$$

When summing over i, the contributions of the chain structure $\mathcal{H}_0[\mathbf{r}]$ and the excluded volume interaction $\mathcal{H}_u[\mathbf{r}]$, (2.1), exactly cancel. The center-of-mass motion is not influenced by the internal interactions of the chain as long as the external potential $V(\mathbf{r})$ vanishes. This also can be proven diagrammatically: All contributions proportional to u^n in $\mathcal{G}_{\mathrm{cm}}(\mathbf{q}, t)$ exactly vanish. Only contributions of v or mixed ones of u and v in higher order perturbation theory survive. The v terms are the fingerprints of the non-Markovity of the process after ensemble averaging: In the construction of the generating functional in (3.6) we have used the fact that in a single sample of the random potential ensemble the process is Markovian. Translational invariance, however, is broken. So we cannot expect free diffusion

as in (4.9). After ensemble averaging, translational invariance is restored up to effects of order of ξ, but now the process is non Markovian in time as can be seen, e.g., in (3.20) or (3.43). The center-of-mass motion probes the non-Markovity of the ensemble-averaged process.

This effect will now be calculated to one-loop order. We use a mixed notation of segment Green's functions $G_{ij}(t)$, (3.40), and the center-of-mass mode function $G_0(t)$, (3.31), for $k=0$. Due to (3.22), (3.40), and (4.3) they are related by

$$\frac{G_0(t)}{N} = \frac{1}{N}\sum_{i=1}^{N} G_{ij}(t) = \frac{1}{d}\langle \mathbf{R}_{\mathrm{cm}}[\mathbf{r}(t_0)]\, \mathbf{R}_{\mathrm{cm}}[\mathbf{r}(t+t_0)]\rangle_0$$
$$= \xi^2 \mathrm{e}^{-\Gamma|t|/\xi^2} \tag{4.12}$$

The last expression was derived by additional use of (3.27) and (3.41). It is consistent with (4.6) and (4.7).

By means of (3.42)–(3.45) we find for $t \geqslant t'$

$$\begin{aligned}
&\mathscr{G}_{\mathrm{cm}}(\mathrm{q}, t-t')_{0+1} \\
&\quad = \overline{\mathscr{X}_1[\mathbf{h}_{\mathrm{cm}}, \tilde{\mathbf{h}} \equiv 0]}_{0+1} \\
&\quad = \exp[-\mathrm{q}^2\xi^2(1-\mathrm{e}^{-\Gamma(t-t')/\xi^2})] \\
&\qquad \times \Bigg(1 + \frac{v}{2}\int_{\mathrm{p}} l^d \sum_{i,j=1}^{N} \int_{-\infty}^{\infty} \gamma^2\, dt_1\, dt_2 \\
&\qquad \times \left\{-\mathrm{p}^2 i R_{ji}(t_2-t_1) + \frac{\mathrm{pq}}{N}[iR_0(t'-t_1) - iR_0(t-t_1)]\right\} \\
&\qquad \times \left\{-\mathrm{p}^2 i R_{ij}(t_1-t_2) - \frac{\mathrm{pq}}{N}[iR_0(t'-t_2) - iR_0(t-t_2)]\right\} \\
&\qquad \times \exp\{-\mathrm{p}^2 D_{ij}(t_1-t_2)\} \\
&\qquad \times \exp\left\{-\frac{\mathrm{pq}}{N}[G_0(t'-t_1) - G_0(t'-t_2) - G_0(t-t_1) + G_0(t-t_2)]\right\}\Bigg)
\end{aligned} \tag{4.13}$$

The subscript $0+1$ on $\mathscr{G}_{\mathrm{cm}}(\mathrm{q}, t-t')$ denote the zeroth and first order of perturbation theory. In (4.13) $R_{ij}(0)=0$ due to (3.38) has been used, as well as the short-hand notation

$$D_{ij}(t) := \frac{G_{ii}(0) + G_{jj}(0) - 2G_{ij}(t)}{2} = \frac{\langle (\mathrm{r}_i(t+t_0) - \mathrm{r}_j(t_0))^2\rangle_0}{2d} \tag{4.14}$$

We now use the invariance under the exchange $(i, \mathrm{p}, t_1) \leftrightarrow (j, -\mathrm{p}, t_2)$ and the representation of the response functions as products of Θ-functions times derivatives of Green's functions (3.41). We also insert the explicit form of $G_0(t)$, (4.12). We can partially integrate over the smaller one of the times t_1 or t_2 to find for $t > 0$

$$\begin{aligned}
&\mathscr{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1}/\mathscr{G}_{\mathrm{cm}}(\mathrm{q}, t)_0 \\
&\quad = 1 + v \int_{\mathrm{p}} l^d (\mathrm{pq})^2 \int_0^t \Gamma\, d\tau\, \mathrm{e}^{-\Gamma(t-\tau)/\xi^2} \sum_{i,j=1}^{N} \mathrm{e}^{-\mathrm{p}^2 D_{ij}(t)} \int_0^{t-\tau} \Gamma\, d\tau'\, \mathrm{e}^{\mathrm{pq}\,\mathscr{A}(t,\tau,\tau')} \\
&\quad = 1 + \hat{v}\, \mathrm{q}^2 \int_0^t \Gamma\, d\tau\, \mathrm{e}^{-\Gamma(t-\tau)/\xi^2} \int_0^S \frac{ds_i\, ds_j}{4} D_{ij}(\tau)^{-1-d/2} \\
&\qquad \times \int_0^{t-\tau} \Gamma\, d\tau' \left(1 + \frac{\mathrm{q}^2 \mathscr{A}^2(t, \tau, \tau')}{2D_{ij}(\tau)}\right) \mathrm{e}^{\mathrm{q}^2 \mathscr{A}^2(t,\tau,\tau')/(4D_{ij}(\tau))}
\end{aligned} \tag{4.15}$$

where

$$\mathscr{A}(t, \tau, \tau') = \xi^2 (1 - \mathrm{e}^{-\Gamma\tau/\xi^2})(\mathrm{e}^{-\Gamma\tau'/\xi^2} - \mathrm{e}^{-\Gamma(t-\tau-\tau')/\xi^2})$$

In the last expression continuous chain variables are introduced.

In the analysis of this result it is essential to know the long- as well as the short-time properties of the distance function $D_{ij}(t)$, (4.14). They are derived in detail in Appendix A and here will be briefly summarized.

As one can expect from the last expression in (4.14), $D_{ij}(t)$ can be shown to be a monotonously growing function both in time $|t|$ and in the continuous segment distance $|s_i - s_j| = |i-j|\, l^2$. For $t \neq 0$ it is found to be furthermore growing monotonously in the variable $|s_i + s_j - S| = |i+j-N|\, l^2$ as long as s_i, s_j are taken from the physical parameter space $s_i, s_j \in [0, S]$. The form $|s_i + s_j - S|$ obviously has to appear in order to treat both ends of the chain 0, S in a symmetrical way. Thus

$$\begin{aligned}
&\left.\frac{\partial D_{ij}(t)}{\partial |t|}\right|_{s_i, s_j = \mathrm{const}} \geqslant 0 \\
&\left.\frac{\partial D_{ij}(t)}{\partial |(s_i - S/2) \pm (s_j - S/2)|}\right|_{|(s_i - S/2) \mp (s_j - S/2)|,\, |t| = \mathrm{const}} \geqslant 0
\end{aligned} \tag{4.16}$$

For $t = 0$, on the other hand, $D_{ij}(0)$ depends only on $|s_i - s_j|$, and the well-known result of polymer statics

$$D_{ij}(0) = |s_i - s_j|, \qquad D_{ii}(0) = 0 \tag{4.17}$$

is reproduced. The result (4.17) also can be derived directly from $\mathscr{H}_0[\mathbf{r}]$, (2.1).

In Appendix A a clearly distinguished short- and long-time asymptotics of $D_{ij}(t)$ is derived. The time scale discriminating between "short" and "long" is given by the dimensionless time

$$T=\frac{\Gamma|t|}{S}=\frac{\hat{\gamma}|t|}{S^2} \tag{4.18}$$

For $T=\mathcal{O}(1)$, the mean square diffusion length $2d\Gamma|t|$ of the center of mass is of the order of the mean square radius of gyration S. For $T>\mathcal{O}(1)$ the whole volume of the chain most likely has left the previously occupied volume in space.

Also the slowest internal modes $k=\mathcal{O}(1)$ decay on this time scale. (Only the center-of-mass mode has a longer relaxation time, namely $\Gamma|t|=\mathcal{O}(\xi^2)\gg S$.)

For $T\geqslant\mathcal{O}(1)$ we can use the long-time representation of (A.26)

$$D_{ij}(t)=\xi^2(1-\mathrm{e}^{-\Gamma|t|/\xi^2})+Sc_{ij} - S\sum_{k=1}^{\infty}\frac{2}{\pi^2k^2}e^{-\pi^2k^2T/2}\left\{\cos\left(\pi k\frac{s_i-s_j}{S}\right)+\cos\left(\pi k\frac{s_i+s_j}{S}\right)\right\} \tag{4.19}$$

since only few terms of the k sum contribute. Here

$$c_{ij}=\frac{1}{6}+\left(\frac{s_i-S/2}{S}\right)^2+\left(\frac{s_j-S/2}{S}\right)^2, \qquad \frac{1}{6}\leqslant c_{ij}\leqslant\frac{2}{3} \tag{4.20}$$

For $T\gg 1$ the center-of-mass motion (4.6), (4.7) dominates,

$$D_{ij}(t)=\frac{\langle(\mathbf{R}_{\mathrm{cm}}[\mathbf{r}(t)]-\mathbf{R}_{\mathrm{cm}}[\mathbf{r}(0)])^2\rangle_0}{2d}+\mathcal{O}(S)\{1+\mathcal{O}(\mathrm{e}^{-\pi^2T/2})\} \tag{4.21}$$

For $T\leqslant\mathcal{O}(1)$, k modes up to $k^2T=\mathcal{O}(1)$ contribute in (4.19). For $T\to 0$ in particular, the simple result of (4.17) must be recovered. Clearly, therefore a more appropriate representation for $T<1$ needs to be found. Applying Poisson's sum formula, we derive in appendix A that

$$D_{ij}(t)=(2\hat{\gamma}|t|)^{1/2}F\left(\frac{s_i-s_j}{(2\hat{\gamma}|t|)^{1/2}},\frac{s_i+s_j}{(2\hat{\gamma}|t|)^{1/2}},\frac{2S}{(2\hat{\gamma}|t|)^{1/2}}\right) \quad \text{for } (2\hat{\gamma}|t|)^{1/2}\ll\xi^2 \tag{4.22}$$

with

$$F(y,z,\lambda)=|y|+\sum_{\nu=-\infty}^{\infty}(g(y+\nu\lambda)+g(z+\nu\lambda)) \quad \text{for} \quad \lambda=(2/T)^{1/2}\geqslant\mathcal{O}(1) \quad \text{and} \quad \xi^2\gg S \tag{4.23}$$

and

$$g(y)=f(y)-|y|, \qquad f(y)=\frac{e^{-y^2}}{\sqrt{\pi}}+y \operatorname{erf} y$$
$$2f(y)-2yf'(y)-f''(y)=0 \tag{4.24}$$

erf y is the error function. The properties of the functions $g(y)$ and $f(y)$ are discussed in appendix A in detail.

For $T \ll 1$ we have

$$D_{ij}(t)=(2\hat{\gamma}\,|t|)^{1/2}\left\{f\left(\frac{s_i-s_j}{(2\hat{\gamma}\,|t|)^{1/2}}\right)+g\left(\frac{s_i+s_j}{(2\hat{\gamma}\,|t|)^{1/2}}\right)+g\left(\frac{s_i+s_j-2S}{(2\hat{\gamma}\,|t|)^{1/2}}\right)\right.$$
$$\left.+\mathcal{O}(|T|\,e^{-1/(2|T|)})\right\} \tag{4.25}$$

The function $(2\hat{\gamma}\,|t|)^{1/2} f((s_i-s_j)/(2\hat{\gamma}\,|t|)^{1/2})$ is the infinite-chain contribution, which was previously calculated by de Gennes[17] and used by Martinez-Mekler and Moore,[10] though not reduced to the form (4.24).

The g-functions originate from the chain ends. For $t \to 0$ they vanish for almost all s_i+s_j:

$$\lim_{T\to 0}\frac{D_{ij}(t)}{(2\hat{\gamma}t)^{1/2} f((s_i-s_j)/(2\hat{\gamma}t)^{1/2})}=1 \qquad \text{for fixed} \quad s_i+s_j \neq 0, 2S \tag{4.26}$$

The infinite chain function has two well-known limits,

$$(2\hat{\gamma}t)^{1/2} f\left(\frac{s_i-s_j}{(2\hat{\gamma}t)^{1/2}}\right)=\begin{cases}|s_i-s_j| & \text{for} \quad t=0\\ (2\hat{\gamma}t/\pi)^{1/2} & \text{for} \quad s_i=s_j\end{cases} \tag{4.27}$$

The $t \to 0$ limit reproduces the static result (4.17) as required. The $s_i \to s_j$ limit yields the well-known "$t^{1/4}$ law" characteristic for the short-time behavior of the internal displacements of a Rouse chain.

If on the other hand we also are interested in the behavior for larger times, the g-functions must be kept, since they are the precursor of the full function (4.22) or (4.19).

Note that for an infinite chain, which would be described by only the f-function as in (4.25), the cross-over to the $T>1$-behavior never occurs, since the mean square radius of gyration S is infinite. On the other hand, for the center-of-mass motion $\mathcal{G}_{cm}(q, t)$ of a very long but finite chain, $T=\mathcal{O}(1)$ discriminates between short- and long-time asymptotics.[11] It is thus a requirement of consistency to use the full function.

The knowledge of the function $D_{ij}(t)$ allows for the evaluation of $\mathcal{G}_{cm}(q, t)_{0+1}$ (4.15) in appendix B. Special attention is paid to the diver-

gencies of the theory, since these are needed for the renormalization group (RG) analysis, as pointed out in Section 3.4.

Here a short comparison of the relation of polymer statics or polymer dynamics toward field theory seems in place. The transformation from polymer statics to field theory essentially is based on two ingredients: on one hand, the Laplace transformation

$$\int_0^S d\,|s_i - s_j|\; e^{-m\,|s_i - s_j|} e^{-\mathrm{q}^2 D_{ij}(0)} = \frac{1}{m + \mathrm{q}^2} \tag{4.28}$$

maps the static polymer propagator to the field theoretic propagator. On the other hand, in a diagrammatic expansion of polymer statics the chain of length N factorizes in segment distances (4.17), each giving a contribution like (4.28) after the Laplace transformation. The diagrammatic structure of such a perturbation expansion is identical with that of ϕ^4 field theory.

On the one hand, comparing $D_{ij}(0)$ with $D_{ij}(t)$ illustrates that there is no obvious extension of the transformation (4.28) between the free propagators of the static theories to a transformation between polymer dynamics and any renormalizable dynamic field theory. In particular the dynamic polymer "propagators" depend not only on $|s_i - s_j|$ and $|t|$, but also on $|s_i + s_j - S|$. The appearance of the latter term is a consequence of the decomposition of the free chain dynamics into the Fourier modes with respect to the intrinsic metric of the chain. On the other hand, the diagrammatic structure is also quite different. In particular, polymer dynamics "diagrams" do not order according to different segmentations of the total arc-length of the chain, but involve rather an order according to the interaction times, as will be discussed in more detail in the next section.

Now we proceed with calculating ε-poles. Before deriving the one needed for the evaluation of Eq. (4.15), for comparison we first give a typical divergent one-loop diagram of polymer statics:

$$\begin{aligned} \frac{\hat{u}}{S} \int_0^S ds_i\, ds_j\, D_{ij}(0)^{1-d/2} &= \hat{u} \int_0^S \frac{ds_i}{S} \int_{-s_i}^{S-s_i} dm\, |m|^{-1+\varepsilon/2} \\ &= \hat{u}\, S^{\varepsilon/2}\, \frac{4 + \mathcal{O}(\varepsilon)}{\varepsilon} \end{aligned} \tag{4.29}$$

S here is the infrared cutoff. The ultraviolet (UV) divergence at $d \geqslant 4$ could have been expressed as dependent on a UV-cutoff l. In the limit of $l \to 0$ it here appears as an ε pole. The UV singularities, resp. the ε poles, then are absorbed in renormalized parameters as in (3.59) and (3.60).

For the dynamic problem we will use the same RG scheme. The upper critical dimension of the dynamical problem needs to be the same as in the

static problem, since the same couplings are involved and the static substructure has to be reproduced. Comparing the appearance of $D_{ij}(t)^{-1-d/2}$ in (4.15) with $D_{ij}(0)^{+1-d/2}$ in (4.29) at first sight casts doubts on the identity of critical dimensions. This, however, is compensated by the additional time integral. The analysis is done in detail in Appendix B. It turns out that the segment distance $(s_i - s_j)$ needs to be rescaled with the time variable $(2\hat{\gamma}\,|t|)^{1/2}$ as is suggested by (4.22) or (4.25). The divergence then appears in (4.15) in the integration over τ for $\tau = 0$. The basic integration to be compared with (4.29) is

$$\begin{aligned}
&\hat{v}\int_0^{|t|} \Gamma\, d\tau \int_0^S ds_i\, ds_j\, D_{ij}(t)^{-1-d/2} \\
&\quad = \hat{v}\int_0^{|t|} \hat{\gamma}\, d\tau \int_0^S \frac{ds_i}{S} \int_{-s_i/(2\hat{\gamma}\tau)^{1/2}}^{(S-s_i)/(2\hat{\gamma}\tau)^{1/2}} dy\, (2\hat{\gamma}\tau)^{(-2+\varepsilon/2)/2} \\
&\qquad \times F\left(y, \frac{2s_i}{(2\hat{\gamma}\tau)^{1/2}} + y, \frac{2S}{(2\hat{\gamma}\tau)^{1/2}}\right)^{-3+\varepsilon/2} \\
&\quad = \hat{v}\,(\min\{(2\hat{\gamma}\,|t|)^{1/2}, S\})^{\varepsilon/2}\, \frac{4I + \mathcal{O}(\varepsilon)}{\varepsilon}
\end{aligned} \tag{4.30}$$

with

$$I = \int_0^\infty dy\, f(y)^{-3} = 3.587 \tag{4.31}$$

with 3.587 from numerical integration of $f(y)$, (4.24).

The singularity obviously originates from short time differences $\hat{\gamma}\tau \to 0$. Furthermore one can convince oneself (see appendix B) that it is also the short segment differences $|s_i - s_j| \leqslant \mathcal{O}((2\hat{\gamma}\tau)^{1/2})$ which mainly contribute to the singularity.

The full ε-poles of $\mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1}$ (4.15) are derived in Appendix B as

$$\begin{aligned}
\mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t) &= \exp\{-\mathrm{q}^2\xi^2\,(1 - \mathrm{e}^{-\Gamma|t|/\xi^2})\} \\
&\quad \times\left(1 + \hat{v}\,\mathrm{q}^2\Gamma\,|t|\,\mathrm{e}^{-\Gamma|t|/\xi^2}\, \frac{I\,(\min\{(2\hat{\gamma}\,|t|)^{1/2}, S\})^{\varepsilon/2} + \mathcal{O}(\varepsilon)}{\varepsilon}\right)
\end{aligned} \tag{4.32}$$

for $0 < \hat{\gamma}\,|t| < \infty$ and $\xi^2 \gg S$, a result which to the order considered can be written as

$$\begin{aligned}
\mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t) &= \exp\{-\mathrm{q}^2\xi^2(1 - \mathrm{e}^{-\Gamma|t|[1 - I\,\bar{v}(l_R)(1+\mathcal{O}(\varepsilon))/\varepsilon]/\xi^2})\} \\
&\quad + \mathcal{O}(\bar{v}^2)
\end{aligned} \tag{4.33}$$

with l_R, e.g., chosen as $(\min\{(2\hat{\gamma}\,|t|)^{1/2}, S\})^{\varepsilon/2}$ and

$$\bar{v}(l_R)\, l_R^{-\varepsilon} = \hat{v}\, Z_v^{-1}(\bar{u}(l_R), \bar{v}(l_R)) \tag{4.34}$$

Clearly the pole must be absorbed into a renormalization of Γ

$$\Gamma = \Gamma_R(l_R)\left(1 + \frac{I}{\varepsilon}\bar{v}(l_R)\right) \tag{4.35}$$

whereas ξ stays unrenormalized

$$\xi = \xi_R(l_R) \tag{4.36}$$

The latter feature had to be expected, since the system volume ξ^d couples to the center-of-mass position of the polymer, which does not depend on the chain size and thus needs no renormalization.

The ansatz (4.35) is the essence of the renormalizability assumption for the dynamic theory, since it implies that whichever quantity one might calculate, each Γ will be accompanied by the very same ε pole (4.35). For polymer statics parametrized by S and u, resp. w, this structure is proven. For polymer dynamics it now will be tested by calculating another expectation value with a very rich structure already in tree approximation.

5. THE CENTER-OF-MASS MOTION OF TWO CHAINS

In the last section we found the renormalization of ξ and Γ to one-loop order. Even after implementing the known renormalization of the static parameters $\hat{w} = \hat{u} - \hat{v}$ and S (cf. Sections 2.1, 2.2, and 3.4) and under the assumption that the dynamic theory is renormalizable, we still lack information on the renormalization of $\hat{v}$. Furthermore, it is desirable to check the renormalization hypothesis by calculating other quantities besides the center-of-mass motion.

In this section we consider a dynamical two-chain cumulant

$$\mathscr{G}_{12}(\mathrm{q}, t-t') = \overline{\langle \exp\{-i\mathrm{q}(\mathrm{R}_{\mathrm{cm}}^{(1)}[\mathbf{r}(t)] - \mathrm{R}_{\mathrm{cm}}^{(2)}[\mathbf{r}(t')])\}\rangle}^{C} \tag{5.1}$$

with $\mathrm{R}_{\mathrm{cm}}^{(r)}[\mathbf{r}(t)]$ being the position of the center of mass of chain number r at time t. The superscript C for "cumulant" indicates that the two chains are connected either by the excluded-volume coupling $\hat{u}$ or by ensemble averaging and its associated coupling $\hat{v}$.

$\mathscr{G}_{12}(\mathrm{q}, t-t')$ is particularly useful, since it depends on $\hat{v}$ even in zero-loop approximation. It is closely analogous to the second virial coefficient of the static theory, which has been successfully employed to find the renormalization of the excluded-volume coupling $\hat{u}$.[1]

Furthermore, its structure in tree approximation is so complex that calculating this quantity to one-loop order will yield the one-loop renormalization of all the relevant parameters. This allows for consistency checks with polymer statics and the results of renormalizing the center-of-mass motion in the last section.

For calculating properties of n chains, the apparatus of Section 3 needs to be slightly extended to include more than one chain. The free theory consists again of the chain structure $\mathscr{H}_0[\mathbf{r}]$, (2.1), the finite embedding volume $\mathscr{H}_\xi[\mathbf{r}]$, (2.7), and the external fields inserted into the dynamic theory. The external fields, however, consist now of $n \times d \times N$ components. For distinction of the chains a superscript (r) is given to the external fields $h_{i\mu}^{(r)}(\tau)$. The free theory decouples in the chain indices r, i.e., the chains are independent. The excluded-volume interaction u as well as the second moment of the potential v couple the chains.

Accordingly, Eqs. (3.42)–(3.45) are generalized to the generating functional for n chains as

$$\begin{aligned}
&\overline{\mathscr{Z}_n[\mathbf{h}_{(0)}, \tilde{\mathbf{h}}_{(0)}]} \\
&\quad =: \sum_{m=0}^{\infty} \overline{\mathscr{Z}_n[\mathbf{h}_{(0)}, \tilde{\mathbf{h}}_{(0)}]}_m \\
&\quad = \sum_{m=0}^{\infty} \frac{1}{m!} \prod_{\sigma=1}^{m} (-S_{I(\sigma)}^{(n)}) \\
&\qquad \times \exp\left(-\frac{1}{2} \int d\tau\, d\tau' \sum_{r=1}^{n} \sum_{i,j=1}^{N} \sum_{\mu=1}^{d} h_{i\mu}^{(r)}(\tau)_{(m)}\, G_{ij}(\tau-\tau')\, h_{j\mu}^{(r)}(\tau')_{(m)} \right) \\
&\qquad \times \exp\left(-\int d\tau\, d\tau' \sum_{r=1}^{n} \sum_{i,j=1}^{N} \sum_{\mu=1}^{d} h_{i\mu}^{(r)}(\tau)_{(m)}\, R_{ij}(\tau-\tau')\, \tilde{h}_{j\mu}^{(r)}(\tau')_{(0)} \right)
\end{aligned} \tag{5.2}$$

with

$$\begin{aligned}
S_{I(\sigma)}^{(n)} = \frac{1}{2} \int_{\mathrm{p}_\sigma}^{*} l^d \sum_{r_\sigma, r'_\sigma=1}^{n} \sum_{i_\sigma, j_\sigma=1}^{N} \int \gamma^2\, dt_\sigma\, dt'_\sigma \\
\times \left\{ u \frac{\delta(t_\sigma - t'_\sigma)}{\gamma} \left[\mathscr{R}(r_\sigma, i_\sigma, \mathrm{p}_\sigma, t_\sigma) + \mathscr{R}(r'_\sigma, j_\sigma, -\mathrm{p}_\sigma, t'_\sigma) \right] \right. \\
\left. - v\, \mathscr{R}(r_\sigma, i_\sigma, \mathrm{p}_\sigma, t_\sigma)\, \mathscr{R}(r'_\sigma, j_\sigma, -\mathrm{p}_\sigma, t'_\sigma) \right\}
\end{aligned} \tag{5.3}$$

and

$$\mathscr{R}(r, i, \mathrm{p}, t) = \int d\tau \sum_{j=1}^{N} \sum_{\mu=1}^{d} (p_\mu h_{j\mu}^{(r)}(\tau)_{(m)})\, iR_{ji}(\tau - t) \tag{5.4}$$

and the fields

$$h_{i\mu}^{(r)}(\tau)_{(m)} = h_{i\mu}^{(r)}(\tau)_{(0)} + \sum_{\sigma=1}^{m} p_{\sigma\mu} [\delta_{rr_\sigma}\delta_{ii_\sigma}\delta(\tau - t_\sigma) - \delta_{rr'_\sigma}\delta_{ij_\sigma}\delta(\tau - t'_\sigma)] \tag{5.5}$$

The two-chain function (5.1) can now be expressed as

$$\begin{aligned}\mathscr{G}_{12}(\mathrm{q}, t) = {} & \overline{\mathscr{X}_2\left[h_{i\mu}^{(r)}(\tau)_{(0)} = -\frac{q_\mu}{N}(\delta_{r1}\delta(\tau - t) - \delta_{r2}\delta(\tau - t')), 0\right]} \\ & - \overline{\mathscr{X}_1\left[h_{i\mu}^{(r)}(\tau)_{(0)} = -\frac{q_\mu}{N}\delta_{r1}\delta(\tau - t), 0\right]} \\ & \times \overline{\mathscr{X}_1\left[h_{i\mu}^{(r)}(\tau)_{(0)} = \frac{q_\mu}{N}\delta_{r2}\delta(\tau - t'), 0\right]}\end{aligned} \tag{5.6}$$

The free theory yields the time-independent result

$$\overline{\mathscr{X}_1\left[h_{i\mu}^{(r)}(\tau)_{(0)} = -\frac{q_\mu}{N}\delta_{r1}\delta(\tau - t), 0\right]}_0 = \mathrm{e}^{-\mathrm{q}^2\xi^2/2} \tag{5.7}$$

and in this approximation

$$\mathscr{G}_{12}(\mathrm{q}, t - t')_0 = 0 \tag{5.8}$$

The first-order perturbation theory $\mathscr{G}_{12}(\mathrm{q}, t - t')_1$ contributes nonvanishing, connected diagrams, the tree approximation. These diagrams are proportional to either $\hat{u}$ or $\hat{v}$. The second-order perturbation theory $\mathscr{G}_{12}(\mathrm{q}, t - t')_2$ yields one-loop diagrams. These diagrams contain the information about the one-loop renormalization of $\hat{u}$ and $\hat{v}$.

The tree approximation is evaluated in Appendix C. In the continuous chain limit the result takes the form

$$\begin{aligned}\mathscr{G}_{12}(\mathrm{q}, t)_1 = {} & \xi^{-d}\frac{1}{2}\int_0^S ds_i\, ds_j\, \mathrm{e}^{-\mathrm{q}^2 S c_{ij}} \\ & \times \left[\hat{v} - \hat{u}\,\mathrm{e}^{-\mathrm{q}^2\Gamma|t|}(1 + \mathrm{q}^2\Gamma|t|) + \mathscr{O}\left(\hat{g}\frac{S}{\xi^2}, \hat{u}\,\mathrm{q}^2\frac{(\Gamma t)^2}{\xi^2}\right)\right]\end{aligned} \tag{5.9}$$

with c_{ij} from (4.20) and

$$\mathcal{O}(\hat{g}) = \mathcal{O}(\hat{u}) + \mathcal{O}(\hat{v}) \tag{5.10}$$

$\mathcal{G}_{12}(q, t)$ has the following properties also beyond the first-order perturbation theory: It vanishes like ξ^{-d} for $\xi \to \infty$. This is understandable from the observation that $\mathcal{G}_{12}(q, t)$ is the cumulant of two independent chains, which should vanish like (embedding volume)$^{-1}$.

As predicted in Section 2.2, for $t = 0$ there is only one coupling $\hat{w} = \hat{u} - \hat{v}$, which determines the static behaviour.

For $t \neq 0$ independent information about $\hat{u}$ and $\hat{v}$ can be gained.

The different time dependence also can be understood: The correlations of the center-of-mass positions of the two polymers due to the mutual repulsion decay exponentially in time, whereas the correlations due to averaging over the quenched random potential are time independent.

Coming back to the technical aspect of the calculation, one observes that the tree approximation contains all parameters $\hat{u}$, $\hat{v}$, Γ, S, and ξ of the theory. The variables $\hat{u}\xi^{-d}$, $\hat{v}\xi^{-d}$, Γ, and S are clearly distinguished by the different coefficients depending on $|t|$ or q^2. The ε poles of higher order perturbation theory thus can be uniquely attributed to these variables. The well-known results on $\hat{u}$ have to be reproduced by the dynamic calculation. Thus from $\hat{u}\xi^{-d}$ and $\hat{v}\xi^{-d}$ also the ε poles belonging to $\hat{v}$ and ξ can be determined.

Now the one-loop contributions $\mathcal{G}_{12}(q, t)_2$ have to be actually determined. The calculation is given in Appendix D. Here only some features of the technique will be outlined. In polymer statics one uses Feynman diagrams to keep track of the different contributions. The contributions are distinguished by the number and the order of the interacting segments along the arc length of the chain. In polymer dynamics, in contrast, one has to keep track of the temporal order of the segment interactions within each chain. This is due to the response prefactors (5.4), which consist of temporal derivatives of parts of the exponential. It is the Θ-functions in $R_{ij}(t)$, (3.41), which determine which parts of the exponential appear in the response prefactors. As in earlier calculations [cf. derivation of (4.15) or Appendix C], the expressions can be made compact by partial integrations in time, starting from the earliest interaction. (This procedure also appears most promising for the renormalizability analysis to all orders of polymer dynamics with excluded-volume interaction, but without quenched random potentials.)

In the course of calculating the expressions of Appendix D, we in fact have used a graphical notation for the temporal order of the segment interactions as a bookkeeping device, but since confusion with usual Feynman

diagrams is predetermined and since also Appendix D does not contain all details of this long but unproblematic calculation, we have refrained from introducing such "diagrams."

We will give here but one intermediate step of the calculation of Appendix D. One basically and before considering temporal order has to distinguish two different types of connected diagrams in second-order perturbation theory: in the $2+2$ diagrams both couplings act between the chains. In the $3+1$ diagrams there is one coupling between the two chains and one coupling is a self-interaction of one chain. $3+1$ is meant to indicate that there are three segments of one and one segment of the other chain involved in the two interactions.

Accordingly we split the one-loop expression into

$$\mathscr{G}_{12}(\mathrm{q}, t)_2 = g(\mathrm{q}, t)_{(3+1)} + g(\mathrm{q}, t)_{(2+2)} \tag{5.11}$$

and find in Appendix D

$$\begin{aligned} g(\mathrm{q}, t)_{(3+1)} = \xi^{-d} \frac{1}{4} \int_0^S ds_i\, ds_j\, ds_k\, ds_l\, \mathrm{e}^{-\mathrm{q}^2 S c_{kl}} \\ \times \Big(\{\hat{v} - \hat{u}\, \mathrm{e}^{-\mathrm{q}^2 \Gamma t}\,(1 + \mathrm{q}^2 \Gamma t)\}\, (\hat{u} - \hat{v}) \\ \times D_{ij}(0)^{-d/2} \left(1 - \exp \frac{\mathrm{q}^2 [G_{ik}(0) - G_{jk}(0)]^2}{4 D_{ij}(0)}\right) \\ + \hat{u}\hat{v}\, \mathrm{e}^{-\mathrm{q}^2 \Gamma t}\, 2 \int_0^t \Gamma^2\, d\tau\, d\tau'\, \theta(t - \tau - \tau')\, \Gamma(t - \tau - \tau') \\ \times \mathrm{q}^2 \int \frac{d^d \mathrm{p}}{\pi^{d/2}}\, \mathrm{pq}\, \frac{\partial G_{jk}(\tau')}{\Gamma\, \partial \tau'} \\ \times \mathrm{e}^{-\mathrm{q}^2 D_{ij}(\tau)} - \mathrm{pq}(G_{ik}(\tau + \tau') - G_{jk}(\tau') - \Gamma\tau)] \Big) \end{aligned} \tag{5.12}$$

for the $3+1$ diagrams and $t \geqslant 0$, and

$$\begin{aligned} g(\mathrm{q}, t)_{(2+2)} = \xi^{-d} \int \frac{d^d \mathrm{p}_1\, d^d \mathrm{p}_2}{\pi^{d/2}}\, \delta^d(\mathrm{q} - \mathrm{p}_1 - \mathrm{p}_2) \\ \times \frac{1}{8} \int_0^S ds_i\, ds_j\, ds_k\, ds_l\, \mathrm{e}^{-\mathrm{p}_1 \mathrm{q} S c_{ij} - \mathrm{p}_2 \mathrm{q} S c_{kl}} \\ \times \Big\{ [\hat{v}^2 + (\hat{u} - 2\hat{v})\, \hat{u}\, \mathrm{e}^{-\mathrm{q}^2 \Gamma t}(1 + \mathrm{q}^2 \Gamma t)]\, \mathrm{e}^{\mathrm{p}_1 \mathrm{p}_2 (D_{ik}(0) + D_{jl}(0))} \end{aligned}$$

$$+2\hat{u}^2\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}(\mathrm{p}_1\mathrm{q})(\mathrm{p}_2\mathrm{q})\int_0^t \Gamma\,d\tau\;\Gamma(t-\tau)\,\mathrm{e}^{\mathrm{p}_1\mathrm{p}_2(D_{ik}(\tau)+D_{jl}(\tau))}$$

$$-2\hat{u}\hat{v}\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}\mathrm{q}^2\int_0^t \Gamma\,d\tau\;\Gamma(t-\tau)(\mathrm{p}_1\mathrm{q}\,\mathrm{e}^{\mathrm{p}_1\mathrm{q}\Gamma\tau}+\mathrm{p}_2\mathrm{q}\,\mathrm{e}^{\mathrm{p}_2\mathrm{q}\Gamma\tau})$$

$$\times\,\mathrm{e}^{\mathrm{p}_1\mathrm{p}_2(D_{ik}(\tau)+D_{jl}(0))}$$

$$-\hat{u}\hat{v}\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}\int_0^t \Gamma^2\,d\tau\,d\tau'\;\theta(t-\tau-\tau')[-1+\mathrm{q}^2\Gamma(t-\tau-\tau')]$$

$$\times[(\mathrm{p}_1\mathrm{q})^2\,\mathrm{e}^{\mathrm{p}_1\mathrm{q}\Gamma(\tau+\tau')}+(\mathrm{p}_2\mathrm{q})^2\,\mathrm{e}^{\mathrm{p}_2\mathrm{q}\Gamma(\tau+\tau')}]$$

$$\times\,\mathrm{e}^{\mathrm{p}_1\mathrm{p}_2(D_{ik}(\tau)+D_{jl}(\tau'))}\Bigg\} \tag{5.13}$$

for the $2+2$ diagrams. We suppress corrections of $\mathcal{O}(\hat{g}\,S/\xi^2)$ or of $\mathcal{O}(\hat{u}\mathrm{q}^2(\Gamma t)^2/\xi^2)$.

The ε poles of $\mathcal{G}_{12}(\mathrm{q},t)_2$ are calculated in Appendix E. Comparison of the first term in (5.12) with the tree approximation (5.9) leads us to expect that this loop integral cannot yield ε poles belonging to Γ, while the more complicated time structure of the second term in (5.12) hints at ε poles of dynamic origin. In Appendix E it is in fact found that

$$g(\mathrm{q},t)_{(3+1)}=\xi^{-d}\frac{1}{2}\int_0^S ds_k\,ds_l\,\mathrm{e}^{-\mathrm{q}^2Sc_{kl}}$$

$$\times\Bigg\{[\hat{v}-\hat{u}\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}(1+\mathrm{q}^2\Gamma t)](-\mathrm{q}^2Sc_{kl})\,(\hat{u}-\hat{v})\,\frac{S^{\varepsilon/2}+\mathcal{O}(\varepsilon)}{\varepsilon}$$

$$-\hat{u}\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}(\mathrm{q}^2\Gamma t)^2\,\hat{v}\;\frac{I\,(\min\{S^2,2\hat{\gamma}t\})^{\varepsilon/4}+\mathcal{O}(\varepsilon)}{\varepsilon}\Bigg\} \tag{5.14}$$

with the constant I from Eq. (4.30).

Of the four terms in (5.13) only the first one contributes an ε pole, and Appendix E yields

$$g(\mathrm{q},t)_{(2+2)}=\xi^{-d}\frac{1}{2}\int_0^S ds_k\,ds_l\,\mathrm{e}^{-\mathrm{q}^2Sc_{kl}}$$

$$\times\Bigg[2\hat{v}^2\,\frac{S^{\varepsilon/2}+\mathcal{O}(\varepsilon)}{\varepsilon}$$

$$+\hat{u}\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}(1+\mathrm{q}^2\Gamma t)\,(2\hat{u}-4\hat{v})\,\frac{S^{\varepsilon/2}+\mathcal{O}(\varepsilon)}{\varepsilon}\Bigg] \tag{5.15}$$

The S dependence of the expressions (5.9), (5.14), and (5.15) requires a brief reconsideration. By means of (4.20) and with the substitution $\mathbf{i} = (s_i - S/2)/S$ etc., we find that

$$\int_0^S ds_i\, ds_j\, \mathrm{e}^{-\mathrm{q}^2 S c_{ij}} = S^2 \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, \mathrm{e}^{-\mathrm{q}^2 S(1/6 + \mathbf{i}^2 + \mathbf{j}^2)} \tag{5.16}$$

The complete expression for $\mathscr{G}_{12}(\mathrm{q}, t)$ up to second-order perturbation theory can thus be summarized from (5.9), (5.14), and (5.15) as

$$\begin{aligned}
&\mathscr{G}_{12}(\mathrm{q}, t)_{0+1+2} \\
&\quad = \xi^{-d} \frac{1}{2} S^2 \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, \mathrm{e}^{-\mathrm{q}^2 S(1/6 + \mathbf{i}^2 + \mathbf{j}^2)} \\
&\qquad \times \Big\{ [\hat{v} - \hat{u}\, \mathrm{e}^{-\mathrm{q}^2 \Gamma t}(1 + q^2 \Gamma t)] \\
&\qquad \times \left[1 - \mathrm{q}^2 S \left(\frac{1}{6} + \mathbf{i}^2 + \mathbf{j}^2 \right) (\hat{u} - \hat{v}) \frac{S^{\varepsilon/2} + \mathcal{O}(\varepsilon)}{\varepsilon} \right] \\
&\qquad - \hat{u}\, \mathrm{e}^{-\mathrm{q}^2 \Gamma t} (\mathrm{q}^2 \Gamma t)^2\, \hat{v}\, \frac{I(\min\{S^2, 2\hat{\gamma} t\})^{\varepsilon/4} + \mathcal{O}(\varepsilon)}{\varepsilon} \\
&\qquad + 2\hat{v}^2 \frac{S^{\varepsilon/2} + \mathcal{O}(\varepsilon)}{\varepsilon} + \hat{u}\, \mathrm{e}^{-\mathrm{q}^2 \Gamma t} (1 + \mathrm{q}^2 \Gamma t)\, (2\hat{u} - 4\hat{v}) \frac{S^{\varepsilon/2} + \mathcal{O}(\varepsilon)}{\varepsilon} \Big\}
\end{aligned} \tag{5.17}$$

We now introduce renormalized parameters. The first term in (5.17) together with the minimal subtraction scheme for the ε poles determines uniquely that the chain size has to be renormalized as

$$S = S_R(l_R) \left(1 - \frac{\bar{u}(l_R) - \bar{v}(l_R)}{\varepsilon} \right) \tag{5.18}$$

with the arbitrary length scale $0 < l_R < \infty$. The second term yields

$$\Gamma = \Gamma_R(l_R) \left(1 + \frac{I\, \bar{v}(l_R)}{\varepsilon} \right) \tag{5.19}$$

Using (5.18) once more for S^2, one can read from the third term in (5.17) the required replacement

$$\xi^{-d}\, \hat{v} = \xi_R(l_R)^{-d}\, \bar{v}_R(l_R)\, l_R^{-\varepsilon} \left(1 + \frac{2\bar{u}(l_R) - 4\bar{v}(l_R)}{\varepsilon} \right) \tag{5.20}$$

and from the fourth, that

$$\xi^{-d}\hat{u} = \xi_R(l_R)^{-d}\bar{u}_R(l_R)\, l_R^{-\varepsilon}\left(1+\frac{4\bar{u}(l_R)-6\bar{v}(l_R)}{\varepsilon}\right) \tag{5.21}$$

The renormalized parameters eliminate exactly the ε poles and not more (minimal subtraction).

Now it has to be tested whether the renormalization (5.18)–(5.21) of the parameters as derived from $\mathcal{G}_{12}(q, t)$, (5.17), is consistent with the previous knowledge about the theory.

1. We remark that the renormalization of Γ in (4.35) as derived from $\mathcal{G}_{cm}(q, t)$, is reproduced in (5.19).

2. We know from polymer statics without disorder[1] that

$$S = S_R(l_R)\left(1-\frac{\bar{u}(l_R)}{\varepsilon}\right), \quad \hat{u} = \bar{u}_R(l_R)\, l_R^{-\varepsilon}\left(1+\frac{4\bar{u}(l_R)}{\varepsilon}\right) \qquad \text{for} \quad \hat{v}\equiv 0 \tag{5.22}$$

The result on S is reproduced by (5.18). The result on $\hat{u}$ is reproduced in (5.21) we additionally use

$$\xi = \xi_R(l_R) \tag{5.23}$$

The latter identity was already derived from $\mathcal{G}_{cm}(q, t)$ in (4.36).

3. In Section 2.2 we recalled that in the limit of zero segment concentration in static expectation values, the theory with quenched potentials can be derived from the theory without random potentials by simply replacing the excluded-volume coupling $\hat{u}$ by the new coupling

$$\hat{w} = \hat{u} - \hat{v} \tag{5.24}$$

The subtraction of ε poles is designed such as to absorb UV divergencies. It should be independent of the system volume. We therefore conclude that

$$S = S_R(l_R)\left(1-\frac{\bar{w}(l_R)}{\varepsilon}\right), \qquad \hat{w} = \bar{w}_R(l_R)\, l_R^{-\varepsilon}\left(1+\frac{4\bar{w}(l_r)}{\varepsilon}\right) \tag{5.25}$$

must hold in a renormalizable theory. We furthermore know that the renormalization of S and $\hat{w}$ to all orders has to be independent of the coupling $\hat{v}$, because $\hat{v}$ does not play any separate role besides $\hat{w}$ within this static subtheory. The requirement (5.25) is fulfilled by the result of $\mathcal{G}_{12}(q, t)$, as can be derived from (5.18), (5.20), (5.21), (5.23), and the definition (5.24).

The last observation leads us to rather consider $\hat{v}$ and $\hat{w}$ as the basic couplings of the theory. Then $\hat{v}$ reads

$$\hat{v} = \bar{v}_R(l_R)\, l_R^{-\varepsilon}\left(1 - \frac{2\bar{v}(l_R) - 2\bar{w}(l_R)}{\varepsilon}\right) \tag{5.26}$$

Equations (5.19) and (5.23)–(5.26) make up the full one-loop renormalization scheme of the theory, which now already has passed a number of consistency tests.

6. THE INTERNAL MODES OF A CHAIN

The quantity to be discussed in this section is

$$\begin{aligned}
&\mathscr{G}_{\text{int}}(i, j, \mathrm{q}, t - t') \\
&\quad = \overline{\langle \exp\{-i\mathrm{q}[r_i(t) - \mathrm{r}_j(t')]\}\rangle} \\
&\quad = \overline{\mathscr{Z}_1[h_i(\tau)_{(0)} = -\mathrm{q}(\delta_{ii}\delta(t-\tau) - \delta_{ij}\delta(t'-\tau)), \tilde{\mathbf{h}}_{(0)} \equiv 0]}
\end{aligned} \tag{6.1}$$

There are several physical as well as technical reasons to calculate this quantity: Since the motion of the free chain can be decomposed into the center-of-mass motion and the internal modes, $\mathscr{G}_{\text{int}}$ contains information on the chain motion independent of $\mathscr{G}_{\text{cm}}$. Both $\mathscr{G}_{\text{cm}}(\mathrm{q}, t)$ and $\mathscr{G}_{\text{int}}(i, j, \mathrm{q}, t)$ are typical quantities measured in Monte Carlo simulations.

Furthermore, the expectation value (6.1) is of experimental interest because $\sum_{i,j=1}^{N} \mathscr{G}_{\text{int}}(i, j, \mathrm{q}, t)$ is the dynamic structure factor, which can be measured in light scattering experiments.[6, 18]

In addition, $\mathscr{G}_{\text{int}}$ plays a double role in the renormalizability analysis: On one hand the renormalization of the parameters of the theory has been determined uniquely in the last section and the calculation of (6.1) serves as a renormalizability test. On the other hand $\exp\{-i\mathrm{q}[\mathrm{r}_i(t) - \mathrm{r}_j(t')]\}$ is a generic insertion appearing in higher order perturbation theory, as can be seen in (3.20). Such terms are included in the formal aparatus of perturbation theory by the additional internal fields (3.45). Higher order loop calculations therefore will always include the renormalization of this structure.

In tree approximation it is easily derived that

$$\begin{aligned}
\mathscr{G}_{\text{int}}(i, j, \mathrm{q}, t - t')_0 &= \exp\left\{-\frac{\mathrm{q}^2}{2}[G_{ii}(0) + G_{jj}(0) - 2G_{ij}(t - t')]\right\} \\
&= \exp[-\mathrm{q}^2 D_{ij}(t - t')]
\end{aligned} \tag{6.2}$$

with the definition (4.14) of $D_{ij}(t)$. The properties of $D_{ij}(t)$ are derived in Appendix B and summarized in Eqs. (4.16)–(4.27). In (4.14) $D_{ij}(t-t')$ is also identified with the expectation value $\langle (\mathrm{r}_i(t)-\mathrm{r}_j(t'))^2\rangle_0$ in tree approximation. Beyond this approximation the identity

$$\overline{\langle (\mathrm{r}_i(t)-\mathrm{r}_j(t'))^2\rangle} = -\triangle_{\mathrm{q}}|_{\mathrm{q}=0}\, \mathscr{G}_{\mathrm{int}}(i,j,\mathrm{q},t-t') \tag{6.3}$$

holds. Equations (6.2)–(6.3) are in close correspondence to Eqs. (4.4)–(4.7).

The one-loop expression for $\mathscr{G}_{\mathrm{int}}(i,j,\mathrm{q},t-t')$ is

$$\begin{aligned}
&\mathscr{G}_{\mathrm{int}}(i,j,\mathrm{q},t-t')_1 \\
&\quad = -\frac{1}{2}\int_{\mathrm{p}} l^d \sum_{k,l=1}^{N} \int_{-\infty}^{\infty} \gamma^2\, dt_1\, dt_1' \\
&\qquad \times \Big\{ u\frac{\delta(t_1-t_1')}{\gamma}\left[\mathscr{R}(k,\mathrm{p},t_1)+\mathscr{R}(l,-\mathrm{p},t_1')\right] \\
&\qquad - v\,\mathscr{R}(k,\mathrm{p},t_1)\,\mathscr{R}(l,-\mathrm{p},t_1')\Big\} \\
&\qquad \times \exp\{-\mathrm{q}^2 D_{ij}(t-t') - \mathrm{p}^2 D_{kl}(t_1-t_1')\} \\
&\qquad \times \exp\{-\mathrm{pq}[G_{ik}(t-t_1) - G_{il}(t-t_1') - G_{jk}(t'-t_1) + G_{jl}(t'-t_1')]\}
\end{aligned} \tag{6.4}$$

with the response factors

$$\begin{aligned}
\mathscr{R}(k,\mathrm{p},t_1) &= \mathrm{pq}[iR_{ik}(t-t_1) - iR_{jk}(t'-t_1)] - \mathrm{p}^2 iR_{lk}(t_1'-t_1) \\
\mathscr{R}(l,-\mathrm{p},t_1') &= -\mathrm{pq}[iR_{il}(t-t_1') - iR_{jl}(t'-t_1')] - \mathrm{p}^2 iR_{kl}(t_1-t_1')
\end{aligned} \tag{6.5}$$

Calculation as in previous sections yields in continuous chain variables

$$\begin{aligned}
&\mathscr{G}_{\mathrm{int}}(i,j,\mathrm{q},t)_1 \\
&\quad = \mathrm{e}^{-\mathrm{q}^2 D_{ij}(t)}\frac{1}{4}\int_0^S ds_k\, ds_l \\
&\qquad \times \Big\{ -(\hat{u}-\hat{v})\,|s_k-s_l|^{-d/2}\left[\exp\left(\frac{\mathrm{q}^2\mathscr{B}_{ijkl}(0,0,t)^2}{4\,|s_k-s_l|}\right)-1\right. \\
&\qquad \left. -\int_0^t d\tau\,\frac{\partial}{\partial t}\exp\left(\frac{\mathrm{q}^2\mathscr{B}_{ijkl}(0,\tau,t-\tau)^2}{4\,|s_k-s_l|}\right)\right] \\
&\qquad + \hat{v}\,\mathrm{q}^2\int_0^t d\tau\, d\tau'\,\theta(t-\tau-\tau')\,D_{kl}(\tau)^{-(d+2)/2}\frac{\partial D_{jk}(\tau')}{\partial \tau'}\frac{\partial D_{il}(t-\tau-\tau')}{\partial(t-\tau-\tau')} \\
&\qquad \times\left(1+\frac{\mathrm{q}^2\mathscr{B}_{ijkl}(\tau,\tau',t-\tau')^2}{2D_{kl}(\tau)}\right)\exp\left(\frac{\mathrm{q}^2\mathscr{B}_{ijkl}(\tau,\tau',t-\tau')^2}{4D_{kl}(\tau)}\right)\Big\}
\end{aligned} \tag{6.6}$$

with the abbreviation

$$\mathscr{B}_{ijkl}(t, t', t'') = D_{ik}(t'') - D_{il}(t'' - t) - D_{jk}(t') + D_{jl}(t + t') \tag{6.7}$$

Note that in contrast to the center-of-mass motion, which only depends on $\hat{v}$, here also a term proportional to $\hat{u} - \hat{v} = \hat{w}$ appears, which is obviously a static contribution. For the internal modes the self-interaction of the chain does play a role.

Which ε poles do we have to find in the integrals (6.6) if renormalizability holds? Exactly those which will be eliminated by introduction of the renormalized parameters in the tree approximation (6.2). This requires that we reconsider the functional dependence of $D_{ij}(t)$ on the parameters of the theory. From the representation (4.22) for $(2\hat{\gamma}t)^{1/2} \ll \xi^2$ we derive

$$D_{ij}(t) = S\,(2T)^{1/2}\, F\left(\frac{\mathbf{i}-\mathbf{j}}{(2T)^{1/2}}, \frac{\mathbf{i}+\mathbf{j}+1}{(2T)^{1/2}}, \frac{2}{(2T)^{1/2}}\right) = S\,\mathscr{F}(\mathbf{i}, \mathbf{j}, T) \tag{6.8}$$

with

$$T = \frac{\hat{\gamma}t}{S^2} = \frac{\Gamma t}{S} \qquad \text{and} \qquad \mathbf{i} = \frac{s_i - S/2}{S} \in \left[-\frac{1}{2}, \frac{1}{2}\right] \tag{6.9}$$

The segment variables **i, j** are invariant under renormalization. t is the physically measurable time and thus also invariant. The parameters S and Γ are subject to renormalization. By means of (5.19) and (5.25) $D_{ij}(t)$ reads in renormalized parameters

$$D_{ij}(t) = S_R(l_R)\left(1 - \frac{\bar{w}(l_R)}{\varepsilon}\right) \mathscr{F}\left(\mathbf{i}, \mathbf{j}, \frac{\Gamma_R(l_R)\,t}{S_R(l_R)} \frac{1 + I\bar{v}(l_R)/\varepsilon}{1 - \bar{w}(l_R)/\varepsilon}\right) \tag{6.10}$$

Inserting this in (6.2) yields

$$\begin{aligned} &\mathscr{G}_{\text{int}}(i, j, \mathrm{q}, t)_0 \\ &\quad = \exp\left[-\mathrm{q}^2 S_R \mathscr{F}\left(\mathbf{i}, \mathbf{j}, \frac{\Gamma_R t}{S_R}\right)\right] \\ &\qquad \times \left[1 + \left(\frac{\bar{w}(l_R)}{\varepsilon} - \frac{I\bar{v}(l_R) + \bar{w}(l_R)}{\varepsilon} \frac{\partial}{\partial \ln t}\right) \mathrm{q}^2 S_R \mathscr{F}\left(\mathbf{i}, \mathbf{j}, \frac{\Gamma_R t}{S_R}\right) + \mathcal{O}(\bar{g}^2)\right] \end{aligned} \tag{6.11}$$

In a renormalizable theory, the ε poles of (6.6) must exactly eliminate the ε poles of (6.11). Then (6.6) needs to have the following ε poles:

$$\mathscr{G}_{\text{int}}(i,j,\mathrm{q},t)_1 = \mathrm{e}^{-\mathrm{q}^2 D_{ij}(t)}\left(-\frac{\bar{w}(l_R)}{\varepsilon}+\frac{I\bar{v}(l_r)+\bar{w}(l_R)}{\varepsilon}\frac{\partial}{\partial \ln t}\right)\mathrm{q}^2 D_{ij}(t) \times[1+\mathcal{O}(\varepsilon)] \tag{6.12}$$

In Appendix F it is shown that the singularities of $\mathscr{G}_{\text{int}}(i,j,\mathrm{q},t)_1$ actually do have this involved structure. We conclude that the internal modes are also renormalizable within the RG framework derived in the last section.

7. THE RG FLOW OF THE PARAMETERS AND THE RENORMALIZED DIFFUSION CONSTANT

7.1. The One-Loop Results for the Parameters

The one-loop analysis of the three expectation values in Sections 4–6 consistently and in agreement with the static theory results in the renormalized parameters (5.19) and (5.23)–(5.26).

For the two couplings we found in (5.25) and (5.26)

$$\hat{w} = l_R^{-\varepsilon}\bar{w}(l_R)\left[1+\frac{4\bar{w}(l_R)}{\varepsilon}+\mathcal{O}(\bar{w}^2(l_R))\right] \tag{7.1}$$

$$\hat{v} = l_R^{-\varepsilon}\bar{v}(l_R)\left[1-\frac{2\bar{v}(l_R)}{\varepsilon}+\frac{2\bar{w}(l_R)}{\varepsilon}+\mathcal{O}(\bar{g}^2(l_R))\right] \tag{7.2}$$

One specific unrenormalized theory defined by $\hat{v}$ and $\hat{w}$ corresponds to a one-dimensional manifold of renormalized theories parametrized by the length scale l_R. Since $\hat{v}$ and $\hat{w}$ are invariant under variation of l_R, an infinitesimal variation of l_R yields

$$\frac{d\ln \bar{w}(l_R)}{d\ln l_R} = \varepsilon - 4\bar{w}(l_R)+\mathcal{O}(\bar{w}^2(l_R)) \tag{7.3}$$

$$\frac{d\ln \bar{v}(l_R)}{d\ln l_R} = \varepsilon + 2\bar{v}(l_R)-2\bar{w}(l_R)+\mathcal{O}(\bar{g}^2(l_R)) \tag{7.4}$$

called the RG flow of the couplings. The other two parameters S and Γ are expressed in renormalized couplings as

$$S = S_R(l_R)\left[1 - \frac{\bar{w}(l_R)}{\varepsilon} + \mathcal{O}(\bar{w}^2(l_R))\right] \tag{7.5}$$

$$\Gamma = \Gamma_R(l_R)\left[1 + \frac{I\bar{v}(l_R)}{\varepsilon} + \mathcal{O}(\bar{v}^2(l_R), \bar{v}(l_R)\,\bar{w}(l_R))\right] \tag{7.6}$$

with $I = 3.587$ (4.31).

The invariance of the unrenormalized parameters S and Γ under variation of l_R leads to the RG flow equations for $S_R(l_R)$ and $\Gamma_R(l_R)$:

$$\frac{d \ln S_R(l_R)}{d \ln l_R} = \bar{w}(l_R) + \mathcal{O}(\bar{w}^2(l_R)) \tag{7.7}$$

$$\frac{d \ln \Gamma_R(l_R)}{d \ln l_R} = -I\bar{v}(l_R) + \mathcal{O}(\bar{v}^2(l_R), \bar{v}(l_R), \bar{w}(l_R)) \tag{7.8}$$

7.2. Beyond One-Loop

More is known about the RG flows (7.3), (7.4), (7.7), and (7.8) than was derived from the one-loop calculation of the last section.

As outlined in Sections 2.1 and 2.2, the parameters $S_R(l_R)$ and $\bar{w}(l_R)$ belong to a closed substructure. Therefore the corrections already have been written as $\mathcal{O}(\bar{w}^2(l_R))$, since the RG flow of S_R and $\bar{w}$ cannot depend on $\bar{v}$. The $(\bar{w}, S_R)$ substructure is identical with that of polymer statics and thus also with $(n=0)$-ϕ^4-theory, which are known to be renormalizable theories. The RG flow of $S_R(l_R)$ and $\bar{w}(l_R)$ has been calculated to five-loop order.[3] Here we only recall the general structure. In a minimal subtraction scheme

$$\frac{d \ln \bar{w}(l_R)}{d \ln l_R} = \varepsilon + F_w(\bar{w}(l_R)) \tag{7.9}$$

holds generally, with $F_w(\bar{w})$ an analytic function of $\bar{w}$ and independent of ε. For any $\varepsilon > 0$ and for any start value $\bar{w}(l) > 0$, a growing l_R lets $\bar{w}(l_R)$ flow to the stable fixed point w^*, which has been determined from $\varepsilon + F_w(w^*) = 0$ as $w^* = 0.364$ in $d = 3$.[22),4] For $\bar{w}(l) = 0$, $w^* = 0$ is a fixed point, but an unstable one. For $\bar{w}(l) < 0$, there is no fixed point and also the connection of the model with actual polymers breaks down.

[3] Four loop, ref. 19; β function in five loop, ref. 20; correction, ref. 21.

[4] For the redefinition of u^* see ref. 23.

The flow of $S_R(l_R)$, (7.7), is known to higher accuracy, too. With the notations

$$\frac{d \ln S_R(l_R)}{d \ln l_R} =: 2 - \frac{1}{\nu(\bar{w}(l_R))}, \qquad S_R(l_R) =: n_R(l_R)\, l_R^2 \tag{7.10}$$

it follows that

$$-\nu(\bar{w}(l_R)) \frac{d \ln n_R(l_R)}{d \ln l_R} = 1 \tag{7.11}$$

In minimal subtraction, $\nu(\bar{w}(l_R))$ is a function of $\bar{w}(l_R)$ only and independent of ε.

For $l_R \gg l$ two asymptotic behaviors coincide: On one hand, the contributions of all irrelevant operators in the RG sense actually vanish, or the distinguished manifold $\mathcal{M}$ as introduced in the introduction is reached. On the other hand, $\bar{w}(l_R)$ reaches its fixed point w^* for any $\bar{w}(l) > 0$. Then one can be sure that on one hand all three-segment interactions, etc., contribute only in a negligible way, whereas on the other hand $\nu(\bar{w}(l_R))$ can be replaced by the constant $\nu(w^*)$ and exponentiated. This yields an anomalous power law at the fixed point

$$\frac{d}{dl_R} (n_R(l_R)^{\nu(w^*)}\, l_R) = 0 \qquad \text{or} \qquad n_R(l_R) \propto l_R^{-1/\nu(w^*)} \tag{7.12}$$

with the exponent $\nu(0) = 0.5$ and $\nu(w^* = 0.364) = 0.588$ in $d = 3$.

A special case of (7.12) will be needed below. Compare two parametrizations of a model on scales l and l_R, $l_R > l$. Suppose that on the scale l the model is sufficiently described by the parameters on the distinguished manifold $\mathcal{M}$. Suppose further that $\bar{w}(l) \approx w^*$. Denote $n_R(l) =: N$, with N proportional to the true microscopic chain length. Let the scale l_R be fixed by the condition $n_R(l_R) = 1$. Then

$$l_R = N^{\nu(w^*)} l \tag{7.13}$$

The nontrivial power law (7.13) originates from the coincidence of the two asymptotic behaviors discussed above. This coincidence, however, is not necessary. The condition for a universal law is the decay of irrelevant perturbations for $l_R \gg l$. "Universality" means that only few relevant parameters on the manifold $\mathcal{M}$ determine the behavior at large length scales. If additionally a fixed point of the coupling is reached, the universal law is a power law. If not, the universal law takes a more involved form.

This leads us to discuss the coupling $\bar{v}(l_R)$, which in fact does not have a fixed point in physical parameter space.

Some knowledge about $\bar{v}(l_R)$ is due to another mapping, which yields further information about the flow of $\bar{v}(l_R)$ beyond (7.4): By evaluating appropriate limits of two-chain quantities such as $\mathscr{G}_{12}(q, t)$, (5.1), it has been possible to prove the identity of the flow of the couplings $\bar{v}$ and $\bar{w}$ with the couplings of a special ternary polymer system.[2] Ternary systems consist of two types of polymers a and b. Segments of polymer type a interact through a coupling $\bar{u}_{aa}$, etc. The flow of the coupling $\bar{w}$ can be shown to be identical with the flow of the coupling $\bar{u}_{aa}$ or $\bar{u}_{bb}$. The flow of $\bar{v}$ can then be identified with the flow of $-\bar{u}_{ab}$, provided the system is symmetric: $\bar{u}_{aa} \equiv \bar{u}_{bb}$. The equilibrium properties of ternary polymer systems can be expressed by a renormalizable theory. Therefore we know that the substructure spanned by the parameters $\bar{v}$, $\bar{w}$, and S_R is renormalizable. The general structure of the RG flow of $\bar{v}$ is similar to (7.9), namely

$$\frac{d \ln \bar{v}(l_R)}{d \ln l_R} = \varepsilon + F_v(\bar{v}(l_R), \bar{w}(l_R)) \tag{7.14}$$

with $F_v(\bar{v}, \bar{w})$ being analytic in $\bar{v}$ and $\bar{w}$ and independent of ε if a minimal subtraction scheme is used. The function is known to three-loop order:[25]

$$\begin{aligned}\frac{d \ln \bar{v}(l_R)}{d \ln l_R} = \varepsilon &+ 2\bar{v} - 2\bar{w} + 2\bar{v}^2 - 6\bar{v}\bar{w} + \frac{5}{2}\bar{w}^2 + [3 + 3\zeta(3)]\,\bar{v}^3 \\ &- [18 + 12\zeta(3)]\,\bar{v}^2\bar{w} + \left[\frac{91}{8} + 18\zeta(3)\right]\bar{v}\bar{w}^2 \\ &- \frac{111}{8}\bar{w}^3 + \mathcal{O}(\bar{g}^4)\end{aligned} \tag{7.15}$$

Not for any fundamental reasons, but only to simplify the notation, below we will restrict the analysis to $\bar{w}(l_R) = w^*$ at a fixed point. In this case (7.14) has the form

$$\left.\frac{d \ln \bar{v}(l_R)}{d \ln l_R}\right|_{w^*} = \frac{\alpha(w^*)}{\nu(w^*)}\,[1 + \bar{v}(l_R)\, f_{w^*}(\bar{v}(l_R))] \tag{7.16}$$

with

$$\frac{\alpha(w^*)}{\nu(w^*)} := \frac{2 - \nu(w^*)\, d}{\nu(w^*)} \equiv \varepsilon + F_v(0, w^*) \tag{7.17}$$

$$\frac{\alpha(w^*)}{\nu(w^*)}\,\bar{v} f_{w^*}(\bar{v}) := F_v(\bar{v}, w^*) - F_v(0, w^*) \tag{7.18}$$

The appearance of $\alpha(w^*)/\nu(w^*)$, (7.17), as the leading term in an expansion in $\bar{v}$ has been proven in refs. 24 and 25. Here $\alpha(w^*) = 2 - d\nu(w^*)$ is the "specific heat" exponent at the fixed point w^*. The quantity α/ν also can be interpreted as the Hausdorff dimension of the intersection points of two mutually noninteracting but self-repelling polymers.

Fixing $\bar{u}_{aa} = \bar{u}_{bb} = w^*$ in the ternary polymer system, $\bar{u}_{ab}(l_R)$ will reach a stable fixed point $u^*_{ab}(w^*)$ for large l_R for any starting value $\bar{u}_{ab}(l) > 0$. Another fixed point is $\bar{u}_{ab} = 0$; it is unstable under growing l_R for arbitrary $\bar{u}_{ab}(l) \neq 0$. For any negative start value of $\bar{u}_{ab}(l)$ no fixed point is known.

A negative value of $\bar{u}_{ab}$ maps to a positive value of $\bar{v}$. Because v was introduced as the second moment of the potential, it is always positive, i.e., the coupling is attractive. $\bar{v}$ therefore does not reach a fixed point for nonvanishing disorder.

Now we follow the logic outlined in the introduction. We started from any model on a truly microscopic scale. Increasing the basic length scale of the description, the theory approaches the manifold $\mathcal{M}$ of relevant parameters. Assume that the manifold is approximately reached on the scale l. For $l_R \geqslant l$ the treatment of only the relevant parameters as contained in our model represents the full physical system, and the RG flow within $\mathcal{M}$ is described by Eqs. (7.9)–(7.11) and (7.14). The RG flow within $\mathcal{M}$ can now be integrated safely over a finite interval of l, independent of the existence of a fixed point. It gives a universal result. Such universal laws can also be expressed as invariances. The RG flow of $\bar{v}$, (7.16), can be expressed as such an invariance:

$$\frac{d}{dl_R}\left[\bar{v}(l_R)\, l_R^{-\alpha/\nu} \exp\left(-\int_0^{\bar{v}(l_R)} \frac{f_{w^*}(v)\, dv}{1 + v f_{w^*}(v)}\right)\right] = 0 \tag{7.19}$$

(If $1 + v^* f_{w^*}(v^*) = 0$ for any $v^* > 0$, (7.19) would describe the usual RG flow of $\bar{v}(l_R)$ to a nontrivial fixed point v^*, but there is none for $v > 0$.) Equation (7.19) is equivalent to (7.16) and holds generally in the realm of perturbation theory. Its evaluation up to now is restricted by the fact that only few terms (7.15) of the expansion of $f_{w^*}(v)$ are known. Therefore the theory can be quantitatively evaluated only for $\bar{v}(l_R) \leqslant \mathcal{O}(1)$. This restriction, however, is of only technical origin.

Under the conditions of Eq. (7.14), i.e., if the theory can be appropriately described by only the relevant parameters N and $\bar{w}(l) \approx w^*$ on the length scale l, and if furthermore l_R is fixed by $n_R(l_R) = 1$, one finds

$$\bar{v}(l)\, N^{\alpha} = \bar{v}(N^{\nu} l) \exp\left(-\int_{\bar{v}(l)}^{\bar{v}(N^{\nu} l)} \frac{f_{w^*}(v)\, dv}{1 + v f_{w^*}(v)}\right) \tag{7.20}$$

If corrections from $\bar{v}(l)\, f_{w^*}(\bar{v}(l)) \ll 1$ can be neglected, $\bar{v}(l)$ can be set equal to zero in the exponential on the right-hand side of (7.20), and $\bar{v}(N^n l)$ is a function only of $\bar{v}(l)\, N^\alpha$

$$\bar{v}(N^n l) = \mathscr{V}(\bar{v}(l)\, N^\alpha) \tag{7.21}$$

This identity can be found by inversion of (7.20). The inverse function $\mathscr{V}(v)$ exists, because (7.16) does not have a fixed point.

We now finally integrate the RG flow of $\Gamma_R(l_R)$, (7.8). Since Γ is the essence of a dynamical theory, no mappings to static theories are available. The only information we have beyond (7.8) is, that all terms of perturbation theory proportional to $u^n v^0$, n arbitrary, vanish; cf. (4.11). Since the coupling w comes in by annealed averaging, the argument (4.11) can be extended to the inclusion of fluctuating potentials: All contributions to Γ vanish which are proportional to $\bar{w}^n \bar{v}^0$, n arbitrary. If the theory is renormalizable, we therefore expect in general, that

$$\frac{d \ln \Gamma_R(l_R)}{d \ln l_R} = -I\, \bar{v}(l_R)\; g(\bar{v}(l_R), \bar{w}(l_R)) \tag{7.22}$$

with $g(0, 0) = 1$ and $g(\bar{v}, \bar{w})$ analytical in both parameters and independent of ε in minimal subtraction. It is not known, whether $g(\bar{v}, \bar{w})$ actually depends on $\bar{w}$, but it appears likely from preliminary calculations.

By means of (7.16) and of the monotony of $\bar{v}(l_R) > 0$ as a function of l_R, the invariance

$$\frac{d}{d\bar{v}(l_R)} \left[\Gamma_R(l_R)\; \exp\left(\frac{v}{\alpha} \int_0^{\bar{v}(l_R)} \frac{I\, g(v, w^*)\, dv}{1 + v f_{w^*}(v)} \right) \right] = 0 \tag{7.23}$$

at a fixed point w^* of $\bar{w}$ is derived. Thus $\Gamma_R(l_R)$ at a fixed point w^* of $\bar{w}$ is uniquely determined by $\Gamma_R(l)$, $\bar{v}(l)$, and $\bar{v}(l_R)$. There is no explicit dependence on l_R or ε left. In the limits of Eqs. (7.14) and (7.20), $\Gamma_R(N^n l)$ can be written as

$$\Gamma_R(N^n l) = \Gamma_R(l)\; \exp\left(-\frac{v}{\alpha} \int_{\bar{v}(l)}^{\bar{v}(N^n l)} \frac{I\, g(v, w^*)\, dv}{1 + v f_{w^*}(v)} \right) \tag{7.24}$$

Further information about $g(\bar{v}, \bar{w})$ can only be gained from physical considerations. From the proportionality of $\Gamma_R(N^n l)$ to the diffusion constant for a chain of length N on one hand and from the expectation, that longer chains should be stronger bound to local energy valleys on the other hand, we always expect

$$\frac{d \ln \Gamma_R(l_R)}{d \ln l_R} < 0 \qquad \text{for} \quad \bar{v}(l_R) > 0 \tag{7.25}$$

This leads to analyze finally the renormalized diffusion constant.

7.3. The Renormalized Diffusion Constant

First results on the evaluation of the center-of-mass motion can be found in ref. 11. Especially two asymptotes are natural objects of examination. The short-time asymptotics will be quantitatively analyzed and compared with Monte Carlo data in a forthcoming paper.[12] Here we discuss the long-time asymptotics of the center-of-mass motion.

In tree approximation, diffusion was defined as the limit of $\Gamma|t| \ll \xi^2$ in (4.9), i.e., as the motion in seemingly infinite space. For the rest of the section we will let $\xi \to \infty$. The internal motion of the chain defines another time scale $T = \Gamma|t|/S$, (4.18). For $v \neq 0$ the internal modes couple to the center of mass mode, and there is a short- and a long-time asymptotics of the center-of-mass motion, distinguished by $T = \mathcal{O}(1)$. Following convention we call the limit of $T \gg 1$ diffusion. Discussing diffusion, we therefore concentrate on the mean square displacement $R^2(t)$, (4.6), of the center of mass within time t in the limits of $\xi^2 \gg \Gamma|t| \gg S$. Equations (4.6), (4.9), (4.15), (4.30), and Appendix B lead to

$$\begin{aligned} R^2(t)_{0+1} &= 2d\Gamma|t| \left(1 - \hat{v} \int_0^t \Gamma \, d\tau \, \frac{t-\tau}{t} \int_0^S \frac{ds_i \, ds_j}{4} D_{ij}(t)^{-3+\varepsilon/2}\right) \\ &= 2d\Gamma|t| \left\{1 - \hat{v} \, S^{\varepsilon/2} \left[\frac{I}{\varepsilon} + \mathscr{A}\left(\frac{\Gamma|t|}{S}, \varepsilon\right)\right]\right\} \end{aligned} \tag{7.26}$$

with $\mathscr{A}(T, \varepsilon)$ analytic in ε and

$$\begin{aligned} \mathscr{A}(T, \varepsilon) = &\frac{1}{4} \int_0^T d\tau \, (2\tau)^{-1+\varepsilon/4} \frac{t-\tau}{t} \\ &\times \int_0^1 dx \int_{-x/\sqrt{2\tau}}^{(1-x)/\sqrt{2\tau}} dy \, F\left(y, \frac{2x}{\sqrt{2\tau}} + y, \frac{2}{\sqrt{2\tau}}\right)^{-3+\varepsilon/2} \\ &- \frac{1}{4} \int_0^1 d\tau \, 2^{-1} \tau^{-1+\varepsilon/4} \\ &\times \int_0^1 dx \int_{-\infty}^{\infty} dy \, f(y)^{-3}, \qquad f(y) = F(y, \infty, \infty) \end{aligned} \tag{7.27}$$

In renormalized parameters, $R^2(t)$ reads with $T_R := \Gamma_R(l_R)\,|t|/S_R(l_R)$ and with $n_R(l_R)$ from (7.10),

$$\begin{aligned} R^2(t) = 2d\Gamma_R(l_R)\,|t| \Big\{1 - \bar{v}(l_R) \Big(I \frac{n_R(l_R)^{\varepsilon/2} - 1}{\varepsilon} + n_R(l_R)^{\varepsilon/2} \mathscr{A}(T_R, \varepsilon)\Big) \\ \times [1 + \mathcal{O}(\bar{g}(l_R))]\Big\} \end{aligned} \tag{7.28}$$

l_R is a free parameter of the renormalized theory, which we now fix by the condition $n_R(l_R)=1$, i.e., we choose l_R of the order of the size of the polymer coil. With this choice the analytical remainder reduces considerably, and we have to evaluate

$$R^2(t)=2d\Gamma_R(l_R)\,|t|\{1-\bar{v}(l_R)\,\mathscr{A}(T_R,\varepsilon)\,[1+\mathcal{O}(\bar{g}(l_R))]\} \tag{7.29}$$

The diffusion constant in particular is given by

$$\begin{aligned} D_{v,w,N,\gamma} &:= \lim_{t\to\infty}\frac{R^2(t)}{t} \\ &= 2d\Gamma_R(l_R)\,\{1-\bar{v}(l_R)\,\mathscr{A}(\infty,\varepsilon)\,[1+\mathcal{O}(\bar{g}(l_R))]\} \\ &\quad \text{for} \quad n_R(l_R)=1 \end{aligned} \tag{7.30}$$

Numerical integration yields $\mathscr{A}(\infty,\varepsilon)=-2.218+\mathcal{O}(\varepsilon)$. (Were we to evaluate $\mathscr{A}(\infty,\varepsilon)$ for $\varepsilon=1$, our procedure would follow exactly that under the name of renormalization in $d=3$ as introduced by Schloms and Dohm.[22])

If the theory is also renormalizable beyond one-loop order, one expects the diffusion constant in general to be given by

$$D_{v,w,N,\gamma}=2d\Gamma_R(l_R)\,\{1-\bar{v}(l_R)\,a(\bar{v}(l_R),\bar{w}(l_R),\varepsilon)\} \qquad \text{for} \quad n_R(l_R)=1 \tag{7.31}$$

$\bar{v}(l_R)\,a(\bar{v}(l_R),\bar{w}(l_R),\varepsilon)$ is again the analytical remainder left over after minimal subtraction of the ε poles. Accordingly $a(\bar{v}(l_R),\bar{w}(l_R),\varepsilon)$ is analytical in all its arguments and $a(0,\bar{w}(l_R),\varepsilon)=\mathscr{A}(\infty,\varepsilon)$. It is not known, whether $a(\bar{v}(l_R),\bar{w}(l_R),\varepsilon)$ in general depends on $\bar{w}$, but it appears likely from some preliminary calculations.

Equations (7.30), resp. (7.31), if renormalizability holds beyond one-loop order, are the central result of the RG calculation of the diffusion constant. They are completely general, i.e., not restricted to a fixed point of $\bar{w}$ a small value of $\bar{v}(l)$ or alike. They give the full nonlinear scaling behavior of the diffusion constant.

Let us now relate the parameters on the scale l_R again to those on a smaller intermediate scale l with parameters also on the distinguished manifold $\mathscr{M}$. Let $\bar{w}(l)\approx w^*$ and define $n_R(l)=:N$. Then Eqs. (7.14) and (7.24) can be used to rewrite (7.31) as

$$\begin{aligned} D_{v,w^*,N,\gamma} &= 2d\frac{\bar{\gamma}_R(l)}{N}\exp\left(-\frac{\nu}{\alpha}\int_0^{\bar{v}(N^\nu l)}\frac{I\,g(v,w^*)\,dv}{1+v f_{w^*}(v)}\right) \\ &\quad\times[1-\bar{v}(N^\nu l)\,a(\bar{v}(N^\nu l),w^*,\varepsilon)] \end{aligned} \tag{7.32}$$

with

$$\bar{\gamma}_R(l) = \Gamma_R(l)\, n_R(l)\, \exp\left(\frac{\nu}{\alpha}\int_0^{\bar{v}(l)} \frac{I g(v, w^*)\, dv}{1 + v f_{w^*}(v)}\right) \tag{7.33}$$

or

$$D_{v, w^*, N, \gamma} =: 2d\, \frac{\bar{\gamma}_R(l)}{N}\, \mathscr{D}(\bar{v}(N^\nu l), \varepsilon), \qquad \mathscr{D}(0, \varepsilon) = 1 \tag{7.34}$$

$\gamma_R(l) = \Gamma_R(l)\, n_R(l)$ is the segment diffusion constant on the scale l [cf. Eqs. (2.10) and (4.8)] and thus is independent of macroscopic parameters such as the total chain length. $\bar{v}(l)$ also is a microscopic parameter. Therefore $\bar{\gamma}_R(l)$ also is microscopic. The N dependence of the diffusion constant is thus the explicit one of Eq. (7.34). Note that it does not simply scale like N^{-x}, $x = \text{const}$, as one would derive near a fixed point of the couplings.

Let us now evaluate the expression to lowest order. Equation (7.20) yields

$$\bar{v}(N^\nu l) = \bar{v}(l)\, N^\alpha + \mathcal{O}(\bar{v}(l)) + \mathcal{O}(\bar{v}(l)\, N^\alpha)^2 \tag{7.35}$$

Using further the notation of Eq. (7.30), the diffusion constant becomes explicitly

$$D_{v, w^*, N, \gamma} = 2d\, \frac{\gamma_R(l)}{N}\, \exp\left[-\frac{\nu}{\alpha} I g(0, w^*)\, \bar{v}(l)\, N^\alpha + \mathcal{O}(\bar{v}(l)) + \mathcal{O}(\bar{v}(l)\, N^\alpha)^2\right]$$
$$\times \left[1 - \mathscr{A}(\infty, \varepsilon)\, \bar{v}(l)\, N^\alpha + \mathcal{O}(\bar{v}(l)) + \mathcal{O}(\bar{v}(l)\, N^\alpha)^2\right] \tag{7.36}$$

with the numbers $I = 3.587$ (4.31), $g(0, 0) = 1$ (7.22), and $\mathscr{A}(\infty, \varepsilon) = -2.218 + \mathcal{O}(\varepsilon)$ (7.30).

We seemingly have reproduced here Machta's predicton of $D \propto N^{-1} \exp[-c\bar{v}(l)\, N^\alpha]$ for the diffusion constant in a random potential.[8] (Machta derives this prediction only for a specific model, but the argument can easily be generalized.) Note, however, the neglected terms. Our concrete result (7.36) is useful only in the limit of $\bar{v}(l)\, N^\alpha < 1$. Machta's result, however, can be challenged, too, because he treats the diffusion process as due to transitions of the whole chain between cells of the size of a chain, and thus neglects all the structure of the problem on scales smaller than the chain size.

Accordingly there is no quantitative prediction for the diffusion constant for $\bar{v}(l)\, N^\alpha \gg 1$. Note, however, that even in this limit the general parametric dependence of Eq. (7.34) prevails if the theory is renormalizable. The result (7.34) or more generally (7.31) is the universal result of a RG analysis.

8. SUMMARY AND CONCLUSION

We have treated the diffusion of long polymer chains through a short-ranged quenched random potential. This model is designed to deal with real local potentials as well as with entropic traps or barriers. It is thus complementary to reptation models, which emphasize rather the topological constraints.

Analyzing this model of polymer diffusion in local random potentials, we have shown that it contains all physically relevant parameters in the limit of long chains, i.e., the relevant parameters in the RG sense. These parameters are the arc length of the noninteracting chain S, the diffusion constant $2d\Gamma$ of the noninteracting chain, the excluded-volume coupling u for the strength of the two-segment repulsion, and the second moment v of the random potential.

Using standard methods of the theory of dynamic critical phenomena, we can formulate the theory as a perturbation expansion in u and v about the limit of a freely diffusing Rouse chain. A naive perturbation expansion in the couplings u and v leads to expressions diverging in four dimensions, and a RG approach has to be implemented. The general framework of these calculations was provided before discussing special correlation functions. The couplings of course play different roles: While u is an instantaneous repulsion, v appears as a time-independent interaction after ensemble averaging over the quenched random potentials. It renders the process non-Markovian. Therefore u and v appear in a characteristically different way in the dynamic diagrams, in contrast to static expectation values, which, if renormalizable, depend only on one coupling $w := u - v$. In dynamic expectation values in particular the center-of-mass motion is unaffected by the excluded-volume coupling u. Only the disorder coupling v couples the internal modes to the center-of-mass mode.

We derived the general structure of the perturbation expansion. An essential ingredient of the calculation is the distance function $D_{ij}(t)$, (4.14), which shows quite nontrivial short-time behavior. As a consequence, the integrals yielding the UV singularities differ considerably from integrals in field theory and polymer statics. Also the "diagrammatic" bookkeeping of perturbation theory varies considerably from polymer statics or field theory.

Having to choose correlation functions for calculation, the center-of-mass motion and the internal modes of a chain are obviously interesting. Furthermore, the correlation of the centers of mass of two different chains is the simplest two-chain quantity that can be considered. The correlation functions of these three quantities are calculated to one-loop order. Renormalizability requires that in every expectation value the same microscopic structure is absorbed into a renormalized parameter. The calculation of the

three correlation functions allows for cross-checks of the renormalizability assumption. This does not prove one-loop renormalizability in the strict mathematical sense, but having the information of our calculation, it is hardly imaginable how a strict proof could fail.

The upper critical dimension necessarily is 4, since polymer statics with the upper critical dimension 4 must be contained in the dynamic theory as a closed substructure. Polymer statics has the two relevant parameters S and u. Another closed renormalizable substructure, the ternary polymer system, could be identified by appropriate limits and mappings.[2] This substructure contains three of the four relevant parameters, namely S, $w := u - v$, and v. No information about the truly dynamic parameter Γ can be gained from any transformations or mappings. It is therefore most reassuring that all three distribution functions give an independent and identical result on the renormalization of Γ.

Consider an arbitrary model in a high-dimensional parameter space on the truly microscopic scale. Renormalizability implies that this microscopic theory can be replaced by a different one on an intermediate length scale, which, if the scale is chosen sufficiently large, can be parametrized by only few relevant parameters. These relevant parameters span the low-dimensional distinguished manifold $\mathcal{M}$. The integration of the RG flow of the relevant parameters within $\mathcal{M}$ yields universal results. At a fixed point of the couplings this universal result would take the form of an anomalous power law. We evaluate the theory at the fixed point w^* of the effective static coupling $w = u - v$. The coupling v, however, does not have a perturbatively reachable, stable fixed point. Integrating the RG flow of the parameters not at a fixed point, but within the attractive manifold $\mathcal{M}$, we find universal nonlinear scaling laws. One can identify certain combinations of the parameters as the scaling variables. Then the general structure of the scaling laws and the concrete results of first-order perturbation theory can be discussed.

This technical basis allows us to investigate universal laws for various physical quantities. Here we have concentrated on the mean square diffusion length of the center of mass and its long-time asymptotics, i.e., the diffusion constant D of a chain. As discussed above, we do not derive a simple power law in the chain length N, but $D \propto N^{-1}\mathcal{D}(\bar{v}(N^{\nu}l), \varepsilon)$. In the limit of $\bar{v}(l) \ll 1$ and $N \gg 1$, $\bar{v}(N^{\nu}l)$ is a unique function of $\bar{v}(l)\, N^{\alpha}$, and to lowest order $\bar{v}(N^{\nu}l) \propto \bar{v}(l)\, N^{\alpha}$. The quenched local potentials correct the Rouse prediction $D \propto N^{-1}$ by a factor which is a function only of the renormalized disorder coupling $\bar{v}(N^{\nu}l)$ evaluated on the scale of the whole chain $N^{\nu}l$. It should be compared with the standard reptation result $D \propto N^{-2}$, which models a Rouse dynamics including topological constraints. (Note that the effect of both, local random potentials and topological constraints, cannot

simply be accounted for by multiplying our result on D by an additional factor of N^{-1}.) If $\bar{v}(N^{\nu}l)$ is sufficiently large, we would expect our random potential diffusion constant to decrease faster than N^{-2}. This would mean that for sufficiently long chains asymptotically the effect of entropic traps or random potentials always would dominate over that of topological constraints as accounted for in reptation models. Unfortunately, our one-loop calculation does not give sufficient information on the functions for large $\bar{v}(N^{\nu}l)$ to decide that question. On the other hand, the functional form of our result and its evaluation to lowest order in the renormalized coupling reproduces Machta's saddle point approximation. The diffusion constant calculated in Machta's approximation clearly dominates over reptation.

Our predictions demand an experimental or numerical test. A numerical test allows for easier discrimination of physical mechanisms. The long-time asymptotics of long chains averaged over quenched disorder, however, is hard to simulate. We therefore also calculated the short-time asymptotics of the center-of-mass motion and compared it to Monte Carlo simulations. We will report on the results in a forthcoming paper.[12]

APPENDIX A. REPRESENTATIONS OF THE FUNCTIONS $G_{ij}(t)$ AND $D_{ij}(t)$

Here we derive the long- and the short-time asymptotics of the segment Green function $G_{ij}(t)$, (3.40),

$$G_{ij}(t) = \langle r_{i\mu}(t+t_0)\, r_{j\mu}(t_0)\rangle_0 = \sum_{k=0}^{N-1} O_{ki} O_{kj} G_k(t) \tag{A.1}$$

and of the distance function $D_{ij}(t)$, (4.14) ($i, j \in \{1, 2, \ldots, N\}$),

$$D_{ij}(t) = \frac{1}{2}\langle (r_{i\mu}(t+t_0) - r_{j\mu}(t_0))^2\rangle_0 = \frac{G_{ii}(0) + G_{jj}(0) - 2G_{ij}(t)}{2} \tag{A.2}$$

Evaluating the last expression in (A.1) with the results (3.22), (3.27), (3.28), and (3.31), the discrete chain Green function emerges as

$$\begin{aligned} G_{ij}(t) = {} & \xi^2 \, \mathrm{e}^{-\Gamma |r|/\xi^2} \\ & + \frac{l^2}{2N} \sum_{k=1}^{N-1} \frac{\cos(2\phi_k(i-j)) + \cos(2\phi_k(i+j-1))}{\sin^2 \phi_k} \\ & \times \mathrm{e}^{-2\gamma |r| \, l^{-2} \sin^2 \phi_k} \end{aligned} \tag{A.3}$$

We use the abbreviations

$$\phi_k = \frac{\pi k}{2N} \quad \text{and} \quad \Gamma = \frac{\gamma}{N} = \frac{\hat{\gamma}}{S} \tag{A.4}$$

For $t=0$, the k sum can be evaluated exactly by induction in $|i-j|$, $|i+j-1| = 0, 1, \ldots$. It yields

$$G_{ij}(0) = \xi^2 + Nl^2 \left[2\left(\frac{N-|i-j|}{2N}\right)^2 + 2\left(\frac{N-|i+j-1|}{2N}\right)^2 - \frac{1}{3} - \frac{1}{6N^2} \right] \tag{A.5}$$

By means of (A.2) and (A.5), $D_{ij}(0)$ turns out to be identical to

$$D_{ij}(0) = |i-j|\, l^2 = |s_i - s_j| \tag{A.6}$$

This is the well known result of the noninteracting ($u=0$) static theory, which also can be derived directly from Eq. (2.1).

In contrast to Eqs. (A.5) and (A.6), the $t \neq 0$ contributions cannot be evaluated analytically in closed form. To simplify the analysis we take the continuous chain limit, first reconsidering the $t=0$ contributions: In the limit $N \to \infty$, $l \to 0$, and $S = Nl^2$ fixed, we replace

$$\sin^2 \phi_k = \sin^2 l^2 \frac{\pi k}{2S} = l^4 \left(\frac{\pi k}{2S}\right)^2 \left[1 + \mathcal{O}\left(l^4 \frac{k^2}{S^2} \right) \right] \tag{A.7}$$

assuming that the eigenmodes k with $kl^2 = \mathcal{O}(S)$ essentially do not contribute to the large-scale properties of interest. With this approximation the k sum in (A.3) reduces for $t=0$ to the tabulated Fourier sum

$$\sum_{k=1}^{\infty} \frac{\cos \pi k s_m / S}{\pi^2 k^2} = \left(\frac{S - s_m}{2S}\right)^2 - \frac{1}{12} \tag{A.8}$$

$s_m = ml^2 \in [0, 2S]$ is a continuous chain variable. The result for $G_{ij}(0)$, including the estimate of the error due to the continuous chain limit, reads

$$G_{ij}(0) = \xi^2 + Sc_{ij} - |s_i - s_j| + \mathcal{O}(l^2) \tag{A.9}$$

with the abbreviation

$$c_{ij} = \frac{1}{6} + \left(\frac{2s_i - S}{2S}\right)^2 + \left(\frac{2s_j - S}{2S}\right)^2, \qquad \frac{1}{6} \leqslant c_{ij} \leqslant \frac{2}{3} \tag{A.10}$$

For $D_{ij}(0)$, Eq. (A.6) is reproduced exactly.

Now, we analyze the time-dependent part. Equations (A.2) and (A.3) yield

$$\begin{aligned}\bar{D}_{ij}(t) &:= D_{ij}(t) - D_{ij}(0) = -G_{ij}(t) + G_{ij}(0) \\ &= \xi^2 (1 - \mathrm{e}^{-\Gamma |t|/\xi^2}) + S [a_t(i-j) + a_t(i+j-1)]\end{aligned} \tag{A.11}$$

with

$$\begin{aligned}a_t(m) &= \frac{1}{2N^2} \sum_{k=1}^{N-1} \frac{\cos 2m\phi_k}{\sin^2 \phi_k} (1 - \mathrm{e}^{-2\gamma |t| l^{-2} \sin^2 \phi_k}) \\ &= \int_0^{|t|} \frac{\Gamma \, d\tau}{S} \sum_{k=1}^{N-1} \cos 2m\phi_k \, \mathrm{e}^{-2\gamma\tau l^{-2} \sin^2 \phi_k}\end{aligned} \tag{A.12}$$

Γ is defined in (A.4). In (4.10), $2d\Gamma |t|$ was determined as the free ($v = 0$) mean square displacement of the center of mass of a polymer within time t. The quantity $2dS$ is the mean square end-to-end distance of the noninteracting chain ($u = 0 = v$). Consider now the exponent of (A.12) in the continuous chain limit:

$$\frac{2\gamma\tau}{l^2} \sin^2 \phi_k = \begin{cases} k^2 \dfrac{\pi^2}{2} \dfrac{\Gamma\tau}{S} \left[1 + \mathcal{O}\left(\dfrac{k^2}{S^2} l^4 \right) \right], & k = \mathcal{O}(1) \\ \dfrac{2\hat{\gamma}\tau}{l^4} \mathcal{O}(1), & k = \mathcal{O}(N) \end{cases} \tag{A.13}$$

For $\hat{\gamma}\tau \gg \mathcal{O}(l^4)$ the approximation (A.7) is justified, because the contributions of the modes with $k = \mathcal{O}(N)$ are exponentially suppressed. Taking the continuous chain limit, (A.12) takes the form

$$a_t(m) = \int_0^{\Gamma |t|/S} d\bar{\tau} \sum_{k=1}^{\infty} \cos \pi k \frac{s_m}{S} \, \mathrm{e}^{-(\pi^2/2) k^2 \bar{\tau}} \tag{A.14}$$

(We thus exclude the microscopic time scale $\hat{\gamma}\tau \ll \mathcal{O}(l^4)$ together with the microscopic length scale l from our analysis.) The function (A.14) depends only on the dimensionless time

$$T := \frac{\Gamma |t|}{S} = \frac{\hat{\gamma} |t|}{S^2} = \frac{\gamma |t|}{N^2 l^2} \tag{A.15}$$

and on the segments measured on the scale of the whole chain $s_m/S = m/N$.

For $T \geqslant \mathcal{O}(1)$ the temporal integral can be evaluated to give back a form of the structure (A.3)

$$a_t(m) = \sum_{k=1}^{\infty} \frac{\cos(\pi k s_m/S)}{\pi^2 k^2/2} (1 - \mathrm{e}^{-(\pi^2/2) k^2 T}) \tag{A.16}$$

The first part of the sum is the $t=0$ contribution [Eq. (A.8)]. The T-dependent part of the k sum for $T \geqslant \mathcal{O}(1)$ converges quickly due to the exponential suppression of the terms with $k^2 T \gg 1$.

For $T \ll 1$, however, (A.16) cannot easily evaluated. In particular, an expansion in powers of T does not exist. Here the Poisson sum formula is useful

$$\sum_{k=-\infty}^{+\infty} \mathrm{e}^{-\pi^2 k^2 t + 2\pi i k x} = \int_{-\infty}^{+\infty} dk \sum_{\nu=-\infty}^{+\infty} \mathrm{e}^{2\pi i k \nu}\, \mathrm{e}^{-\pi^2 k^2 t + 2\pi i k x}$$

$$= \sum_{\nu=-\infty}^{+\infty} \frac{1}{(\pi t)^{1/2}}\, \mathrm{e}^{-(x+\nu)^2/t} \tag{A.17}$$

since it allows us to evaluate (A.14) as

$$a_t(m) = \frac{-T}{2} + \sum_{\nu=-\infty}^{+\infty} \int_0^T \frac{d\tau}{(2\pi\tau)^{1/2}}\, \mathrm{e}^{-(2\nu + s_m/S)^2/2\tau}$$

$$= \frac{-T}{2} + (2T)^{1/2} \sum_{\nu=-\infty}^{+\infty} g\left(\frac{2\nu + s_m/S}{(2T)^{1/2}}\right) \tag{A.18}$$

with

$$g(y) = \int_0^1 \frac{ds}{2(\pi s)^{1/2}}\, \mathrm{e}^{-y^2/s} = |y| \int_{|y|}^{\infty} \frac{dt}{\sqrt{\pi} t^2}\, \mathrm{e}^{-t^2} \tag{A.19}$$

$$= \frac{\mathrm{e}^{-y^2}}{\sqrt{\pi}} + y \operatorname{erf} y - |y| \tag{A.20}$$

$$= -\frac{\mathrm{e}^{-y^2}}{\sqrt{\pi}} \sum_{n=1}^{\infty} \left(\frac{-1}{y^2}\right)^n \frac{\Gamma(n+1/2)}{\Gamma(1/2)} \tag{A.21}$$

$\Gamma(x)$ is the Γ function and erf denotes the error function

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x dt\, \mathrm{e}^{-t^2}, \qquad \operatorname{erf}(-x) = -\operatorname{erf} x, \quad \operatorname{erf} \infty = 1 \tag{A.22}$$

Since the function $g(y)$ will be central for the further calculation, we list some of its properties:

$$g(-y) = g(y), \qquad 0 = g(\infty) \leqslant g(y) \leqslant g(0) = \frac{1}{\sqrt{\pi}} \tag{A.23}$$

$$g(y) = \mathcal{O}\left(\frac{e^{-y^2}}{y^2}\right) \quad \text{for } |y| \gg 1, \qquad \frac{\partial g(y)}{\partial |y|} = -1 + \operatorname{erf} |y| \leqslant 0 \tag{A.24}$$

Using (A.24), it is found that contributions in (A.18) with

$$\left|\frac{2v+s_m/S}{(2T)^{1/2}}\right| = \left|\frac{2vS+s_m}{(2\hat{\gamma}\,|t|)^{1/2}}\right| \gg 1$$

decay exponentially. Accordingly, for $T \ll 1$ only few terms of the v sum contribute. For $T \gg 1$ or for $T \ll 1$ we thus use Eq. (A.16) or Eq. (A.18), respectively.

Returning now to the evaluation of the functions $G_{ij}(t)$ and $D_{ij}(t)$, the following relation holds in the continuous chain limit due to (A.2), (A.9) and (A.10):

$$G_{ij}(t) = \xi^2 + Sc_{ij} - D_{ij}(t) \tag{A.25}$$

Emphasis from here on will be laid on the evaluation of $D_{ij}(t)$.

For $T \gg 1$, Eqs. (A.6), (A.8), (A.10), (A.11), and (A.16) yield

$$D_{ij}(t) = \xi^2\,(1 - \mathrm{e}^{-\Gamma|t|/\xi^2}) + Sc_{ij} - S\sum_{k=1}^{\infty} \frac{2}{\pi^2 k^2}\,\mathrm{e}^{-\pi^2 k^2 T/2}\left[\cos\left(\pi k\,\frac{s_i - s_j}{S}\right) + \cos\left(\pi k\,\frac{s_i + s_j}{S}\right)\right] \tag{A.26}$$

As was to be expected, $D_{ij}(t)$ for $T \gg 1$ approaches [cf. (4.6), (4.7)]

$$\begin{aligned} D_{ij}(t) &= \xi^2\,(1 - \mathrm{e}^{-\Gamma|t|/\xi^2}) + \mathcal{O}(S) \\ &= \tfrac{1}{2}\langle (R_{\mathrm{cm},\mu}(t+t_0) - R_{\mathrm{cm},\mu}(t_0))^2 \rangle_0 + \mathcal{O}(S) \end{aligned} \tag{A.27}$$

i.e., the distance of the segments at large times t is dominated by the distance the center of mass has diffused.

For $T \ll 1$, Eqs. (A.6), (A.11) and (A.18) yield

$$D_{ij}(t) = |s_i - s_j| + \mathcal{O}\left(ST^2\,\frac{S}{\xi^2}\right) + (2\hat{\gamma}\,|t|)^{1/2} \sum_{v=-\infty}^{\infty} \left[g\left(\frac{s_i - s_j + 2vS}{(2\hat{\gamma}\,|t|)^{1/2}}\right) + g\left(\frac{s_i + s_j + 2vS}{(2\hat{\gamma}\,|t|)^{1/2}}\right)\right] \tag{A.28}$$

The contribution of the center-of-mass motion $k = 0$ vanishes up to the correction of the order given above.

On physical grounds [cf. (A.2)] we expect $D_{ij}(t)$ to grow monotonously with t. For $T \leqslant \mathcal{O}(1)$ and $\xi^2 \gg S$ this can be verified by, e.g., inserting the first line of (A.18) into (A.11) and taking the temporal derivative. For $T \geqslant \mathcal{O}(1)$ the same can be derived from the derivative of (A.27). So

$$\left.\frac{\partial D_{ij}(t)}{\partial\,|t|}\right|_{i,j=\mathrm{const}} > 0 \qquad \text{for} \quad t \neq 0 \tag{A.29}$$

Inserting the last equation of (A.24) in (A.28), it is straightforward to derive

$$\left.\frac{\partial D_{ij}(t)}{\partial\,|(s_i-S/2)\pm(s_j-S/2)|}\right|_{|(s_i-S/2)\mp(s_j-S/2)|,\ |t|=\text{const}} > 0 \qquad \text{(A.30)}$$

for $(s_i-S/2)\pm(s_j-S/2)\neq 0$ and $t\neq 0$. The chain parameter $s_i-S/2$ appears for symmetry reasons, since both ends of the chain $s_i=0$, S are indistinguishable.

Using (A.2) or (A.6), we furthermore find

$$D_{ij}(t)>0 \quad \text{for } i\neq j \text{ or } t\neq 0 \qquad \text{and} \qquad D_{ii}(0)=0 \qquad \text{(A.31)}$$

We conclude, that $D_{ij}(t)$ is monotonously growing in each one of the variables $|t|$, $|s_i-s_j|$, and $|s_i+s_j-S|$ starting from $D_{ii}(0)=0$.

The short time behavior can be further approximated by [cf. (A.24), (A.28)]

$$D_{ij}(t)=(2\hat{\gamma}\,|t|)^{1/2}\left[f\left(\frac{s_i-s_j}{(2\hat{\gamma}\,|t|)^{1/2}}\right)+g\left(\frac{s_i+s_j}{(2\hat{\gamma}\,|t|)^{1/2}}\right)+g\left(\frac{s_i+s_j-2S}{(2\hat{\gamma}\,|t|)^{1/2}}\right)\right.$$
$$\left.+\mathcal{O}(|T|\,\mathrm{e}^{-1/(2\,|T|)})\right] \qquad \text{(A.32)}$$

The two g functions represent the chain end contributions, while

$$f(y)=g(y)+|y| \qquad \text{(A.33)}$$

is the contribution which is independent of chain length.

In the limit of infinite chain length $S\to\infty$, $D_{ij}(t)$ reduces to

$$D_{ij}(t)_{S\to\infty}=(2\hat{\gamma}\,|t|)^{1/2}\,f\left(\frac{s_i-s_j}{(2\hat{\gamma}\,|t|)^{1/2}}\right) \qquad \text{(A.34)}$$

This reduced functional dependence $D_{ij}(t)_{S\to\infty}$ was derived previously by de Gennes[17] and was used by Martinez-Mekler and Moore[10] in their calculation. For $T\to 0$ we find

$$\lim_{T\to 0}\left|_{s_i,s_j=\text{const}}\right. \frac{D_{ij}(t)}{(2\hat{\gamma}\,|t|)^{1/2}f((s_i-s_j)/(2\hat{\gamma}\,|t|)^{1/2})}=1$$
$$\text{for almost all} \quad s_i, s_j \qquad \text{(A.35)}$$

Some useful representations of $f(y)$ are

$$f(y)=\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}\,y=\frac{1}{\sqrt{\pi}}+\int_0^{|y|}d\tau\,\mathrm{erf}\,\tau \tag{A.36}$$

$$=\int_{-\infty}^{\infty}\frac{dt}{\sqrt{\pi}}|y+t|\,e^{-t^2}=\frac{e^{-y^2}}{\sqrt{\pi}}\left(1+\sum_{n=1}^{\infty}\frac{(2y^2)^n}{(2n-1)!!}\right) \tag{A.37}$$

It obeys the relations

$$f(-y)=f(y),\qquad \frac{1}{\sqrt{\pi}}=f(0)\leqslant f(y)\leqslant\frac{1}{\sqrt{\pi}}+|y| \tag{A.38}$$

$$\lim_{y\to\pm\infty}\frac{f(y)}{|y|}=1,\qquad \frac{\partial f(y)}{\partial|y|}=\mathrm{erf}\,|y|\geqslant 0,\qquad \left(1-y\frac{\partial}{\partial y}\right)f(y)=\frac{e^{-y^2}}{\sqrt{\pi}}$$
$$2f(y)-2yf'(y)-f''(y)=0 \tag{A.39}$$

For further convenience the functional dependence of the full distance function $D_{ij}(t)$ is summarized in the notation

$$D_{ij}(t)=(2\hat{\gamma}\,|t|)^{1/2}\,F\left(\frac{s_i-s_j}{(2\hat{\gamma}\,|t|)^{1/2}},\frac{s_i+s_j}{(2\hat{\gamma}\,|t|)^{1/2}},\frac{2S}{(2\hat{\gamma}\,|t|)^{1/2}}\right)$$
$$\text{for}\quad (2\hat{\gamma}\,|t|)^{1/2}\ll\xi^2 \tag{A.40}$$

with

$$F(y,z,\lambda)=|y|+\sum_{\nu=-\infty}^{\infty}(g(y+\nu\lambda)+g(z+\nu\lambda)) \tag{A.41}$$

as a convenient form for $\lambda=(2/T)^{1/2}\geqslant\mathcal{O}(1)$ and $\xi^2\gg S$, and

$$F(y,z,\lambda)=\frac{1}{\lambda}+\lambda\left(\frac{1}{3}-\frac{z}{\lambda}+\frac{y^2+z^2}{\lambda^2}\right)$$
$$-\sum_{k=1}^{\infty}\frac{\lambda}{\pi^2k^2}\,e^{-\pi^2k^2/\lambda^2}\left(\cos\frac{\pi k2y}{\lambda}+\cos\frac{\pi k2z}{\lambda}\right) \tag{A.42}$$

for $(S/\xi^2)^{1/2}\ll\lambda=(2/T)^{1/2}\leqslant\mathcal{O}(1)$.

$F(y,z,\lambda)$ has the following useful summetries:

$$F(y,z,\lambda)=F(-y,z,\lambda)=F(y,-z,\lambda)=F(y,z\pm\lambda,\lambda) \tag{A.43}$$

(A.42) seems to violate the last symmetry in (A.43); however see the restriction to $y \in [-\lambda/2, \lambda/2]$, $z \in [0, \lambda]$ of the derivation of (A.8), which is used in (A.42).

Due to (A.41), (A.33), (A.23), and (A.38), the inequality

$$F(y, z, \lambda) \geqslant f(y) \geqslant \frac{1}{\sqrt{\pi}} \tag{A.44}$$

is valid for all y, z and λ.

The factorization of $D_{ij}(t)$ as in (A.40) is very convenient for calculations at $T \ll 1$. For actual evaluations in the case of $T \gg 1$ it is, however, more appropriate to rewrite $D_{ij}(t)$ in the form of (A.26) with the dimensionless system size $\Xi^2 = \xi^2/S$:

$$D_{ij}(t) = S\left[\Xi^2 (1 - e^{-T/\Xi^2}) + \left(\frac{2}{3} + \frac{s_i^2 + s_j^2}{S^2} - \frac{s_i + s_j}{S}\right)\right.$$
$$\left. - \sum_{k=1}^{\infty} \frac{2}{\pi^2 k^2} e^{-\pi^2 k^2 T/2} \left(\cos \frac{\pi k(s_i - s_j)}{S} + \cos \frac{\pi k(s_i + s_j)}{S}\right)\right] \tag{A.45}$$

Here S is factorized out instead of a factor of $(2\hat{\gamma}\,|t|)^{1/2}$.

APPENDIX B. THE ϵ POLES OF $\mathscr{G}_{cm}(q, t)$

Here the divergencies of $\mathscr{G}_{cm}(q, t)_{0+1}$, (4.15), are determined. The expression is given for $t \geqslant 0$ by

$$\mathscr{G}_{cm}(q, t)_{0+1}$$
$$= \exp[-q^2\xi^2(1 - e^{-\Gamma t/\xi^2})]$$
$$\times \left[1 + \hat{v}\, q^2 \int_0^t \Gamma\, d\tau \left(\exp \frac{-\Gamma(t-\tau)}{\xi^2}\right) \int_0^S \frac{ds_i\, ds_j}{4} D_{ij}(\tau)^{-3+\varepsilon/2}\right.$$
$$\left. \times \int_0^{t-\tau} \Gamma\, d\tau' \left(1 + \frac{q^2 \mathscr{A}^2(t, \tau, \tau')}{2D_{ij}(\tau)}\right) \exp \frac{q^2 \mathscr{A}^2(t, \tau, \tau')}{4D_{ij}(\tau)}\right] \tag{B.1}$$

with $\varepsilon = 4 - d$, with $D_{ij}(t)$ from Appendix A and with

$$\mathscr{A}(t, \tau, \tau') = \xi^2 (1 - e^{-\Gamma\tau/\xi^2})(e^{-\Gamma\tau'/\xi^2} + e^{-\Gamma(t-\tau-\tau')/\xi^2}) \tag{B.2}$$

In the limit $\Gamma t/\xi^2 \to 0$ of small times or infinite volume, (B.1) reduces to

$$\lim_{\Gamma t/\xi^2 \to 0} \mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1}$$

$$= \mathrm{e}^{-\mathrm{q}^2 \Gamma t} \left[1 + \hat{v}\, \mathrm{q}^2 \int_0^t \Gamma\, d\tau \int_0^S \frac{ds_i\, ds_j}{4} D_{ij}(\tau)^{-3+\varepsilon/2} \right.$$

$$\left. \times \Gamma(t-\tau) \left(1 + \frac{2\mathrm{q}^2 (\Gamma\tau)^2}{D_{ij}(\tau)} \right) \mathrm{e}^{\mathrm{q}^2(\Gamma\tau)^2/D_{ij}(\tau)} \right] \tag{B.3}$$

ε poles can only originate from zeros of $D_{ij}(\tau)$ in (B.3). Due to the representation (A.40) of $D_{ij}(\tau)$ and the inequality (A.44), the zero can only be hidden in the factor $(2\hat{\gamma}\tau)^{1/2}$. The variables $(s_i - s_j)$ and $(s_i + s_j)$ obviously play different roles, as one sees, e.g., in (A.35). Therefore the substitution is also done in an asymmetric way as $y = |s_i - s_j|/(2\hat{\gamma}\tau)^{1/2}$ and $m = s_i + s_j$. Making further use of (A.43) and expanding the last exponent in (B.1), resp. (B.3), we can write $\mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1}$ for $\Gamma t \leqslant \mathcal{O}(S) \ll \xi^2$ as

$$\mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1}$$

$$= \mathrm{e}^{-\mathrm{q}^2 \Gamma t} \left[1 + \hat{v}\, \mathrm{q}^2 \Gamma t \int_0^t \Gamma\, d\tau \frac{t-\tau}{t} \sum_{n=0}^{\infty} \frac{2n+1}{n!} \left(\frac{\mathrm{q}^2}{4S^2} \right)^n \frac{1}{2} \int_0^S dm \right.$$

$$\left. \times \int_0^{m/(2\hat{\gamma}\tau)^{1/2}} dy\, (2\hat{\gamma}\tau)^{(-2+\varepsilon/2+3n)/2} F\left(y, \frac{m}{(2\hat{\gamma}\tau)^{1/2}}, \frac{2S}{(2\hat{\gamma}\tau)^{1/2}} \right)^{-3+\varepsilon/2-n} \right] \tag{B.4}$$

For $d \leqslant 4$ a divergence of (B.4) only can originate from the $n = 0$ term, which contributes an ε pole. All other expressions are finite. The pole can be evaluated as

$$\int_0^t \Gamma\, d\tau \frac{1}{2} \int_0^S dm \int_0^{m/(2\hat{\gamma}\tau)^{1/2}} dy\, (2\hat{\gamma}\tau)^{-1+\varepsilon/4} F\left(y, \frac{m}{(2\hat{\gamma}\tau)^{1/2}}, \frac{2S}{(2\hat{\gamma}\tau)^{1/2}} \right)^{-3+\varepsilon/2}$$

$$= \frac{1}{4} \int_0^{2\hat{\gamma}t} d\bar{\tau}\, \bar{\tau}^{-1+\varepsilon/4} \int_0^S \frac{dm}{S} \int_0^{m/\sqrt{\bar{\tau}}} dy\, F\left(y, \frac{m}{\sqrt{\bar{\tau}}}, \frac{2S}{\sqrt{\bar{\tau}}} \right)^{-3+\varepsilon/2}$$

$$= \frac{(2\hat{\gamma}t)^{\varepsilon/4}}{\varepsilon} \int_0^{\infty} dy\, f(y)^{-3+\varepsilon/2} + \mathrm{reg}(\varepsilon)$$

$$= \frac{I\,(2\hat{\gamma}t)^{\varepsilon/4}}{\varepsilon} + \mathrm{reg}(\varepsilon) \tag{B.5}$$

where reg(ε) denotes the regular contribution in ε in a consistent ε expansion, and

$$I = \int_0^\infty dy\, f(y)^{-3} = 3.587 \tag{B.6}$$

The number 3.587 is derived by numerical integration of the function $f(y)$, (A.36).

The ε pole (B.5) is independent of t as long as $t > 0$. It can be traced back to the short-time singularity of $D_{ij}(t)$. Since $f(y)$ asymptotically approaches $|y|$ for $|y| \gg 1$ (A.39) and since $f(0) = 1/\sqrt{\pi}$, the integral (B.6) is dominated by contributions of $y = |s_i - s_j|/(2\hat{\gamma}t)^{1/2} \leqslant \mathcal{O}(1)$. The ε pole thus represents a short-segment-distance, short-time divergence. Chain end effects do not appear.

Considering now (B.3) for $T = \Gamma t/S \geqslant \mathcal{O}(1)$, we note that the interval of τ integration also extends to $\Gamma\tau/S \geqslant \mathcal{O}(1)$. On the other hand, for $T = \mathcal{O}(1)$, i.e., $\lambda = \mathcal{O}(1)$, the behavior of $D_{ij}(t)$ changes drastically and a different representation needs to be chosen, as was discussed in Appendix A. We find

$$\lim_{\Gamma t/\xi^2 \to 0} \mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1}$$

$$= \mathrm{e}^{-\mathrm{q}^2\Gamma t}\left[1 + \hat{v}\,\mathrm{q}^2\Gamma t\left(\frac{I(\min\{(2\hat{\gamma}t)^{1/2}, S\})^{\varepsilon/2}}{\varepsilon} + \mathrm{reg}(\varepsilon)\right)\right] \tag{B.7}$$

Let us finally analyze the integrals for $\xi < \infty$. From (B.1) for $\Gamma t = \mathcal{O}(\xi^2)$ we derive after splitting again the τ integration into the parts with $\Gamma\tau/S < 1$ and $\Gamma\tau/S > 1$ and using (A.40), resp. (A.45), that

$$\mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_{0+1} = \mathcal{G}_{\mathrm{cm}}(\mathrm{q}, t)_0\left[1 + \hat{v}\,\mathrm{q}^2\Gamma t\,\mathrm{e}^{-\Gamma t/\xi^2}\left(\frac{I S^{\varepsilon/2}}{\varepsilon} + \mathrm{reg}(\varepsilon)\right)\right] \tag{B.8}$$

$$\text{for} \quad \Gamma t/S > 1$$

(B.7) and (B.8) together yield (4.32).

APPENDIX C. THE TREE APPROXIMATION FOR $\mathcal{G}_{12}(\mathrm{q}, t)$

The tree approximation for $\mathcal{G}_{12}(\mathrm{q}, t)$ is given by the first order of perturbation theory for (5.6). It is evaluated by means of Eqs. (5.2)–(5.5).

The function is a cumulant, therefore there are no disconnected diagrams. Dealing with only one coupling in the tree approximation, this coupling must act between the chains. Therefore the summation $\frac{1}{2}\sum_{r,r'=1}^{2}$ over the chain indices in $S^{(2)}_{I(1)}$, (5.3), can be replaced by

$$\frac{1}{2}\sum_{r,r'=1}^{2}(1-\delta_{rr'})=\sum_{r,r'=1}^{2}\delta_{r1}\delta_{r'2}$$

The evaluation of (5.6) then yields to first order in the couplings

$$\begin{aligned}
&\mathscr{G}_{12}(\mathrm{q}, t-t')_1 \\
&\quad = -\int_{\mathrm{p}} l^d \sum_{i,j=1}^{N} \int_{-\infty}^{\infty} \gamma\, dt_1\, \gamma\, dt_2 \\
&\qquad \times \{u\,\delta(t_1-t_2)\,[\mathscr{R}(i,1,\mathrm{p},t_1)+\mathscr{R}(j,2,-\mathrm{p},t_2)] \\
&\qquad -v(\mathrm{p})\,\mathscr{R}(i,1,\mathrm{p},t_1)\,\mathscr{R}(j,2,-\mathrm{p},t_2)\} \\
&\qquad \times \exp\left(-\frac{1}{2}\int d\tau\, d\tau' \sum_{r=1}^{2}\sum_{i,j=1}^{N}\sum_{\mu=1}^{d} h^{(r)}_{i\mu}(\tau)_{(1)}\, G_{ij}(\tau-\tau')\, h^{(r)}_{j\mu}(\tau')_{(1)}\right)
\end{aligned} \tag{C.1}$$

with the fields

$$h^{(r)}_{i\mu}(\tau)_{(1)} = h^{(r)}_{i\mu}(\tau)_{(0)} + p_\mu(\delta_{ri}\delta_{r1}\delta(\tau-t_1)-\delta_{ij}\delta_{r2}\delta(\tau-t_2)) \tag{C.2}$$

The response operators are

$$\begin{aligned}
\mathscr{R}(i,1,\mathrm{p},t_1) &= -\frac{\mathrm{pq}}{\sqrt{N}}\sum_{i=1}^{N} iR_{ii}(t-t_1)+\mathrm{p}^2 iR_{ii}(0) \\
&= \frac{\mathrm{pq}}{N}\,\theta(t-t_1)\,\frac{\partial G_0(t-t_1)}{\gamma\,\partial t_1}
\end{aligned} \tag{C.3}$$

and

$$\mathscr{R}(j,2,-\mathrm{p},t_2) = \frac{\mathrm{pq}}{N}\,\theta(t'-t_2)\,\frac{\partial G_0(t'-t_2)}{\gamma\,\partial t_2} \tag{C.4}$$

For the last line of (C.3), Eqs. (3.38), (3.41), and (4.12) have been used. The exponential of (C.1) with $h^{(r)}_{i\mu}(\tau)_{(1)}$ from (C.2) takes the form

$$\exp\left(-\frac{1}{2}\int d\tau\, d\tau' \sum_{r=1}^{2}\sum_{i,j=1}^{N}\sum_{\mu=1}^{d} h_{i\mu}^{(r)}(\tau)_{(1)}\, G_{ij}(\tau-\tau')\, h_{j\mu}^{(r)}(\tau')_{(1)}\right)$$

$$= \exp\left\{-\frac{q^2}{N}G_0(0)+\frac{pq}{N}[G_0(t-t_1)+G_0(t'-t_2)]\right.$$

$$\left.-\frac{p^2}{2}[G_{ii}(0)+G_{jj}(0)]\right\}$$

$$= \exp\left\{-q^2\xi^2+\frac{pq}{N}[G_0(t-t_1)+G_0(t'-t_2)]-\frac{p^2}{2}(\xi^2+Sc_{ij})\right\} \qquad \text{(C.5)}$$

where (3.27), (3.31), (4.12), (A.9), and (A.10) have been used.

In summary, after evaluating the θ- and δ-functions of the time variables, we have

$$\mathscr{G}_{12}(q, t-t')_1$$

$$= -e^{-q^2\xi^2}\int_p l^d \sum_{i,j=1}^{N} e^{-p^2(\xi^2+Sc_{ij})}$$

$$\times\left[u\left(\int_{-\infty}^{t} dt_1 \frac{pq}{N}\frac{\partial G_0(t-t_1)}{\partial t_1}+\int_{-\infty}^{t'} dt_1 \frac{pq}{N}\frac{\partial G_0(t'-t_1)}{\partial t_1}\right)\right.$$

$$\times e^{(pq/N)(G_0(t-t_1)+G_0(t'-t_1))}$$

$$-v\int_{-\infty}^{t} dt_1 \frac{pq}{N}\frac{\partial G_0(t-t_1)}{\partial t_1}\int_{-\infty}^{t'} dt_2 \frac{pq}{N}\frac{\partial G_0(t'-t_2)}{\partial t_2}$$

$$\left.\times e^{(pq/N)(G_0(t-t_1)+G_0(t'-t_2))}\right] \qquad \text{(C.6)}$$

The time integrals in the contribution proportional to v easily are evaluated. Making use of (4.12),

$$G_0(t) = N\xi^2\, e^{-\Gamma|t|/\xi^2}$$

we find

$$\int_{-\infty}^{t} d\tau \frac{pq}{N}\frac{\partial G_0(t-\tau)}{\partial \tau}\, e^{(pq/N)\,G_0(t-\tau)} = e^{pq\xi^2} \qquad \text{(C.7)}$$

A completely time-independent term results, as was to be expected.

The time dependence of the coefficient of u is more complicated. With $t \geqslant t'$ some intermediate steps of the calculation are

$$\mathscr{I}_u := \left(\int_{-\infty}^{t'} dt_1 \frac{\mathrm{pq}}{N} \frac{\partial G_0(t-t_1)}{\partial t_1} + \int_{-\infty}^{t'} dt_1 \frac{\mathrm{pq}}{N} \frac{\partial G_0(t'-t_1)}{\partial t_1} \right) \times \mathrm{e}^{(\mathrm{pq}/N)(G_0(t-t_1)+G_0(t'-t_1))}$$

$$= \left(\int_{-\infty}^{t'} dt_1 \frac{\partial}{\partial t_1} + \int_{t'}^{t} dt_1 \frac{\mathrm{pq}}{N} \frac{\partial G_0(t-t_1)}{\partial t_1} \right) \mathrm{e}^{(\mathrm{pq}/N)(G_0(t-t_1)+G_0(t'-t_1))}$$

Integrating the first term and symmetrizing the second by the substitution $t - t_1 \to t_1 - t'$, we find

$$\mathscr{I}_u = \mathrm{e}^{(\mathrm{pq}/N)(G_0(t-t')+G_0(0))} - \mathrm{e}^{2(\mathrm{pq}/N)\,G_0(\infty)} + \int_{t'}^{t} dt_1 \frac{\mathrm{pq}}{N} \frac{\partial (G_0(t-t_1) - G_0(t_1-t'))}{2\,\partial t_1} \mathrm{e}^{(\mathrm{pq}/N)(G_0(t-t_1)+G_0(t_1-t'))}$$

Insertion of Eq. (4.12) yields

$$\mathscr{I}_u = \exp[\mathrm{pq}\,\xi^2(1+\mathrm{e}^{-\Gamma|t-t'|/\xi^2})] + \int_0^{\Gamma|t-t'|/(2\xi^2)} d\tau\, k \cosh\tau\, \mathrm{e}^{(k\cosh\tau)} \tag{C.8}$$

where

$$k = 2\mathrm{pq}\,\xi^2\,\mathrm{e}^{-\Gamma|t-t'|/\xi^2}$$

In the limit of large ξ we find

$$\mathscr{I}_u = \mathrm{e}^{2\mathrm{pq}\xi^2}\,\mathrm{e}^{-\mathrm{pq}\Gamma|t-t'|}(1+\mathrm{pq}\Gamma|t-t'|) + \mathcal{O}\left(\mathrm{pq}\frac{(\Gamma|t-t'|)^2}{\xi^2}\right) \tag{C.9}$$

Inserting these results into (C.6) results in

$$\mathscr{G}_{12}(\mathrm{q},t)_1 = \int \frac{d^d\mathrm{p}\,l^d}{(2\pi)^d}\,\mathrm{e}^{-\xi^2(\mathrm{p}-\mathrm{q})^2} \sum_{i,j=1}^{N} \mathrm{e}^{-\mathrm{p}^2 S c_{ij}} \times \left[v - u\,\mathrm{e}^{-\mathrm{pq}\Gamma|t|}(1+\mathrm{pq}\Gamma|t|) + \mathcal{O}\left(u\,\mathrm{pq}\frac{(\Gamma t)^2}{\xi^2}\right)\right]$$

$$= \xi^{-d}(4\pi)^{-d/2}\,l^d \sum_{i,j=1}^{N} \mathrm{e}^{-\mathrm{q}^2 S c_{ij}} \times \left[v - u\,\mathrm{e}^{-\mathrm{q}^2\Gamma|t|}(1+\mathrm{q}^2\Gamma|t|) + \mathcal{O}\left(g\frac{S}{\xi^2}, u\,\mathrm{q}^2\frac{(\Gamma t)^2}{\xi^2}\right)\right] \tag{C.10}$$

The last expression is evaluated for $S \ll \xi^2$ and $\Gamma |t| \ll \xi^2$. Here $\mathcal{O}(g)$ means $\mathcal{O}(u) + \mathcal{O}(v)$.

APPENDIX D. THE ONE-LOOP DIAGRAMS FOR $\mathcal{G}_{12}(\mathbf{q}, t)$

Here we outline the derivation of the expressions (5.11)–(5.13) for $\mathcal{G}_{12}(\mathrm{q}, t)_2$ from Eqs. (5.2)–(5.6).

$\mathcal{G}_{12}(\mathrm{q}, t)_2$ is defined as the second order of perturbation theory for the function defined in Eq. (5.6). The diagrams have two interaction lines and the two chains are connected by at least one interaction. Accordingly, they can be grouped into two diagram classes, the $3+1$ and the $2+2$ diagrams, as defined before and at Eq. (5.11).

The $3+1$ diagrams will be considered first. They have three interacting segments on one chain and one on the other. Due to the invariance of the observables under renumbering the chains and under $\mathrm{q} \to -\mathrm{q}$ (isotropy of space after ensemble averaging), one can reduce the diagrams to those with three interacting segments on chain no. 1 and one on chain no. 2. The second set of diagrams then simply can be derived by exchange $t \leftrightarrow t'$. One then, however, is not allowed to fix the sign of $t - t'$.

There are two interactions $\sigma = 1, 2$ in (5.2). Fixing $\sigma = 1$ as the self-interaction of chain no. 1 and $\sigma = 2$ as the interaction between the two chains, a factor of 2 is gained. Then $t_1 \geqslant t_1'$ yields another factor of 2 as a coefficient of v in the $\sigma = 1$ interaction. Fixing $r_2 = 1$ and $r_2' = 2$ in the $\sigma = 2$ interaction results in another factor of 2. The $3+1$ diagrams thus are

$$
\begin{aligned}
&g(\mathrm{q}, t-t')_{(3+1)} \\
&\quad = \int_{\mathrm{p}_1, \mathrm{p}_2} l^{2d} \sum_{i,j,k,l=1}^{N} \int_{-\infty}^{\infty} \gamma^4 \, dt_1 \, dt_1' \, dt_2 \, dt_2' \\
&\qquad \times \left\{ \frac{u}{2} \frac{\delta(t_1 - t_1')}{\gamma} \left[\mathscr{R}(1, i, \mathrm{p}_1, t_1) + \mathscr{R}(1, j, -\mathrm{p}_1, t_1') \right] \right. \\
&\qquad \left. - v \, \theta(t_1 - t_1') \, \mathscr{R}(1, i, \mathrm{p}_1, t_1) \, \mathscr{R}(1, j, -\mathrm{p}_1, t_1') \right\} \\
&\qquad \times \left\{ u \frac{\delta(t_2 - t_2')}{\gamma} \left[\mathscr{R}(1, k, \mathrm{p}_2, t_2) + \mathscr{R}(2, l, -\mathrm{p}_2, t_2') \right] \right. \\
&\qquad \left. - v \, \mathscr{R}(1, k, \mathrm{p}_2, t_2) \, \mathscr{R}(2, l, -\mathrm{p}_2, t_2') \right\} \\
&\qquad \times \exp \left\{ -\frac{1}{2} \int d\tau \, d\tau' \sum_{r=1}^{2} \sum_{i,j=1}^{N} h_{i\mu}^{(r)}(\tau)_{(2)} \, G_{ij}(\tau - \tau') \, h_{j\mu}^{(r)}(\tau')_{(2)} \right\} \\
&\qquad + (t \leftrightarrow t')
\end{aligned}
\tag{D.1}
$$

with the fields

$$h_{i\mu}^{(r)}(\tau)_{(2)} = \delta_{r1}\left(-\frac{q_\mu}{\sqrt{N}}\delta(\tau - t) + p_{1\mu}(\delta_{il}\delta(\tau - t_1) - \delta_{ij}\delta(\tau - t_1')) \right.$$
$$\left. + p_{2\mu}\delta_{ik}\delta(\tau - t_2)\right)$$
$$+ \delta_{r2}\left(\frac{q_\mu}{\sqrt{N}}\delta(\tau - t') - p_{2\mu}\delta_{il}\delta(\tau - t_2')\right) \tag{D.2}$$

Inserting these fields, we obtain the exponential function in (D.1) as

$$\exp\left(-\frac{1}{2}\int d\tau\, d\tau' \sum_{r=1}^{2}\sum_{i,j=1}^{N} h_{i\mu}^{(r)}(\tau)_{(2)}\, G_{ij}(\tau - \tau')\, h_{j\mu}^{(r)}(\tau')_{(2)}\right)$$
$$= \exp[-\xi^2(\mathrm{q} + \mathrm{p}_2)^2 - \mathrm{p}_2^2 Sc_{kl} - \mathrm{p}_1^2 D_{ij}(t_1 - t_1')]$$
$$\times \exp\left\{-\frac{\mathrm{p}_1\mathrm{q}}{N}[G_0(t - t_1') - G_0(t - t_1)]\right\}$$
$$\times \exp\left\{-\frac{\mathrm{p}_2\mathrm{q}}{N}[2G_0(0) - G_0(t - t_2) - G_0(t' - t_2')]\right\}$$
$$\times \exp\{-\mathrm{p}_1\mathrm{p}_2[G_{ik}(t_1 - t_2) - G_{jl}(t_1' - t_2)]\} \tag{D.3}$$

Exactly as in the tree approximation (C.10), a factor $\exp[-\xi^2(\mathrm{q} + \mathrm{p}_2)^2]$ appears. For any function $f(p_2)$ which can be expanded in p_2, the following identity holds:

$$\int \frac{d^d\mathrm{p}_2}{(2\pi)^d}\, \mathrm{e}^{-\xi^2(\mathrm{q}+\mathrm{p}_2)^2} f(\mathrm{p}_2) = (4\pi\xi^2)^{-d/2} f(\mathrm{q})\left(1 + \frac{\Delta f(\mathrm{q})}{4\xi^2 f(\mathrm{q})} + \cdots\right) \tag{D.4}$$

As long as the quadratic length $f(\mathrm{q})/\Delta f(\mathrm{q})$ is much smaller than ξ^2, $\exp[-\xi^2(\mathrm{q} + \mathrm{p}_2)^2]$ can be replaced by $(\pi/\xi^2)^{d/2}\,\delta^d(\mathrm{q} + \mathrm{p}_2)$ with an error of the order of $[\Delta f(\mathrm{q})]/[\xi^2 f(\mathrm{q})]$.

The response factors are

$$\mathscr{R}(1, i, \mathrm{p}_1, t_1) = -\frac{\mathrm{p}_1\mathrm{q}}{N} iR_0(t - t_1) - \mathrm{p}_1^2 iR_{ij}(t_1' - t_1) + \mathrm{p}_1\mathrm{p}_2 iR_{ik}(t_2 - t_1)$$

$$\mathscr{R}(1, j, -\mathrm{p}_1, t_1') = \frac{\mathrm{p}_1\mathrm{q}}{N} iR_0(t - t_1') - \mathrm{p}_1^2 iR_{ji}(t_1 - t_1') - \mathrm{p}_1\mathrm{p}_2 iR_{jk}(t_2 - t_1') \tag{D.5}$$

$$\mathscr{R}(1, k, \mathrm{p}_2, t_2) = -\frac{\mathrm{p}_2\mathrm{q}}{N} iR_0(t - t_2) - \mathrm{p}_1\mathrm{p}_2\,[iR_{ki}(t_1 - t_2) - iR_{kj}(t_1' - t_2)]$$

$$\mathscr{R}(2, l, -\mathrm{p}_2, t_2') = -\frac{\mathrm{p}_2\mathrm{q}}{N} iR_0(t' - t_2')$$

Obviously the structure of the second chain is much easier. If $\mathscr{R}(2, l, -p_2, t_2')$ appears with a v interaction, the t_2' dependence can be exactly integrated out to

$$\int_{-\infty}^{\infty} \gamma\, dt_2'\, \mathscr{R}(2, l, -p_2, t_2')\, e^{-(p_2 q/N)(G_0(0) - G_0(t' - t_2'))} = 1 - e^{-p_2 q \xi^2} \tag{D.6}$$

Equation (D.4) cannot directly be applied to the correction $\exp(-p_2 q \xi^2)$ in (D.6) because it contains the scale ξ. If inserted on the left-hand side of (D.4) for $f(p_2)$, however, it vanishes like $\exp(-\frac{3}{4} q^2 \xi^2)$ for $|q|^{-1} \ll \xi$. It thus can be neglected together with the other corrections on the right-hand side of (D.4).

We now decompose $g(q, t)_{(3+1)}$ into

$$\begin{aligned} g(q, t)_{(3+1)} = {} & g(q, t)_{(3+1, v^2)} + g(q, t)_{(3+1, uv)} \\ & + g(q, t)_{(3+1, vu)} + g(q, t)_{3+1, u^2)} \end{aligned} \tag{D.7}$$

Omitting further details of the calculation, we find as the simplest term in the limit of large ξ.

$$\begin{aligned} g(q, t - t')_{(3+1, v^2)} &= -v^2 \left(\frac{l^2}{4\pi\xi^2}\right)^{d/2} \int_{p_1} l^d \sum_{i,j,k,l=1}^{N} e^{-q^2 S c_{kl} - p_1^2 D_{ij}(0)} \\ &\quad \times \int_0^{\infty} d\tau \frac{\partial}{\partial \tau}\, e^{-p_1 q (G_{ik}(\tau) - G_{jl}(\tau))} \\ &= -v^2 \left(\frac{l^2}{4\pi\xi^2}\right)^{d/2} \int_{p} l^d \sum_{i,j,k,l=1}^{N} e^{-q^2 S c_{kl} - p^2 D_{ij}(0)} \\ &\quad \times (1 - e^{-pq(G_{ik}(0) - G_{jl}(0))}) \end{aligned} \tag{D.8}$$

The time dependence is here completely integrated out. Note that in (D.8) the evaluation of the time integral at $\tau = -\infty$ contributes a 1, which is essential for the further calculation. In contrast the evaluation of the time integral at $\tau = -\infty$ did not contribute in (D.6).

If u acts between the chains and v within one chain, one finds

$$\begin{aligned} & g(q, t - t')_{(3+1, vu)} \\ & \quad = vu \left(\frac{l^2}{4\pi\xi^2}\right)^{d/2} \int_{p} l^d \sum_{i,j,k,l=1}^{N} e^{-q^2 S c_{kl} - q^2 \Gamma |t - t'|} \\ & \quad \times \Big[(1 + q^2 \Gamma |t - t'|)\, e^{-p^2 D_{ij}(0)} (1 - e^{-pq(G_{ik}(0) - G_{jk}(0))}) \end{aligned}$$

$$+2\int_0^{|t-t'|} \Gamma^3\, d\tau\, d\tau'\, d\tau''\, \theta(|t-t'|-\tau-\tau'-\tau'')\, \mathrm{q}^2(\mathrm{pq})\, \frac{\partial \mathrm{pq} G_{jk}(\tau')}{\Gamma\, \partial\tau'}$$

$$\times \mathrm{e}^{-\mathrm{p}^2 D_{ij}(\tau) - \mathrm{pq}(G_{ik}(\tau+\tau') - G_{jk}(\tau') - \Gamma\tau)}\Big] \tag{D.9}$$

If the self-interaction of a chain is via u, the factor $\delta(t_1 - t_1')$ facilitates the evaluation of the different time orders and

$$g(\mathrm{q}, t-t')_{(3+1,\,uv)} + g(\mathrm{q}, t-t')_{(3+1,\,u^2)}$$
$$= u\,(v - u(1+\mathrm{q}^2\Gamma\,|t-t'|)\,\mathrm{e}^{-\mathrm{q}^2\Gamma|t-t'|})$$
$$\times \left(\frac{l^2}{4\pi\xi^2}\right)^{d/2} \int_{\mathrm{p}} l^d \sum_{i,j,k,l=1}^{N} \mathrm{e}^{-\mathrm{q}^2 S c_{kl} - \mathrm{p}^2 D_{ij}(0)} (1 - \mathrm{e}^{-\mathrm{pq}(G_{ik}(0) - G_{jk}(0))}) \tag{D.10}$$

After performing the p integration, (D.7)–(D.10) yield Eq. (5.12).

We now proceed to the $2+2$ diagrams. They have both interactions acting between both chains. Fixing $r_1 = 1$, $r_1' = 2$, $r_2 = 1$ and $r_2' = 2$ yields a factor of 4 in the $S^{(2)}_{I(\sigma)}$, (5.3). Explicitly one then derives

$$g(\mathrm{q}, t-t')_{(2+2)} = \frac{1}{2}\int_{\mathrm{p}_1,\mathrm{p}_2} l^{2d} \sum_{i,j,k,l=1}^{N} \int_{-\infty}^{\infty} \gamma^4\, dt_1\, dt_1'\, dt_2\, dt_2'$$
$$\times \left\{ u\,\frac{\delta(t_1 - t_1')}{\gamma}\,[\mathscr{R}(1, i, \mathrm{p}_1, t_1) + \mathscr{R}(2, j, -\mathrm{p}_1, t_1')] \right.$$
$$\left. - v\,\mathscr{R}(1, i, \mathrm{p}_1, t_1)\,\mathscr{R}(2, j, -\mathrm{p}_1, t_1') \right\}$$
$$\times \left\{ u\,\frac{\delta(t_2 - t_2')}{\gamma}\,[\mathscr{R}(1, k, \mathrm{p}_2, t_2) + \mathscr{R}(2, l, -\mathrm{p}_2, t_2')] \right.$$
$$\left. - v\,\mathscr{R}(1, k, \mathrm{p}_2, t_2)\,\mathscr{R}(2, l, -\mathrm{p}_2, t_2') \right\}$$
$$\times \exp[-\xi^2(\mathrm{q} - \mathrm{p}_1 - \mathrm{p}_2)^2 - (\mathrm{p}_1 + \mathrm{p}_2)(\mathrm{p}_1 S c_{ij} + \mathrm{p}_2 S c_{kl})]$$
$$\times \exp\left\{ -\frac{\mathrm{p}_1 \mathrm{q}}{N}\,[2G_0(0) - G_0(t - t_1) - G_0(t' - t_1')] \right\}$$
$$\times \exp\left\{ -\frac{\mathrm{p}_2 \mathrm{q}}{N}\,[2G_0(0) - G_0(t - t_2) - G_0(t' - t_2')] \right\}$$
$$\times \exp\{\mathrm{p}_1 \mathrm{p}_2 [D_{ik}(t_1 - t_2) + D_{jl}(t_1' - t_2')]\} \tag{D.11}$$

The coefficient of ξ^2 again can be evaluated as a δ-function just as in (D.4) if ξ is much larger than all other length scales. Here we find

$$\exp[-\xi^2(q-p_1-p_2)^2] \approx (\pi/\xi^2)^{d/2}\,\delta^d(q-p_1-p_2) \tag{D.12}$$

The response factors are

$$\begin{aligned}\mathscr{R}(1, i, p_1, t_1) &= -\frac{p_1 q}{N} iR_0(t-t_1) + p_1 p_2 iR_{ik}(t_2-t_1)\\ \mathscr{R}(2, j, -p_1, t_1') &= -\frac{p_1 q}{N} iR_0(t'-t_1') + p_1 p_2 iR_{jl}(t_2'-t_1')\end{aligned} \tag{D.13}$$

with the two others found by exchanging indices $1 \leftrightarrow 2$, $i \leftrightarrow k$, and $j \leftrightarrow l$.

As previously, partial integrations in time can be performed to simplify the expressions: Suppose that for the two interactions of the first chain $t_1 \leqslant t_2$ holds. Then

$$\mathscr{R}(1, k, p_2, t_2) = -\frac{p_2 q}{N} iR_0(t-t_2)$$

due to $\theta(0)=0$, (3.38). In this case only $\mathscr{R}(1, i, p_1, t_1)$ and the exponential depend on t_1. Therefore the latter response factor can be written as

$$\mathscr{R}(1, i, p_1, t_1) = -\frac{p_1 q}{N} i\bar{R}_0(t_1-t) + p_1 p_2 i\bar{R}_{ik}(t_1-t_2) + \frac{\partial}{\partial t_1} \tag{D.14}$$

with

$$\bar{R}_0(\tau) = \bar{\theta}(\tau)\frac{\partial G_0(\tau)}{i\gamma\,\partial\tau}, \qquad \bar{\theta}(0)=1, \quad \text{etc.}$$

i.e. $\bar{R}_0(\tau)$ and $\bar{R}_{ij}(\tau)$ are identical with $R_0(\tau)$ and $R_{ij}(\tau)$ except for the value of the θ-function at $\tau=0$. The derivative $\partial/\partial t_1$ in (D.14) acts on the exponential and allows for partial integrations in t_1.

The four types of $2+2$ diagrams are now treated separately according to the couplings appearing, in close analogy to the $3+1$ diagrams, (D.7).

The time dependence of the v^2 terms can be completely integrated out to yield

$$\begin{aligned}&g(q, t-t')_{(2+2,\,v^2)}\\ &\quad= \frac{v^2}{2}\left(\frac{l^2}{4\pi\xi^2}\right)^{d/2}\int_{p_1,p_2} l^d(2\pi)^d\,\delta^d(q-p_1-p_2)\\ &\qquad\times \sum_{i,j,k,l=1}^{N} e^{\{-p_1 q S c_{ij} - p_2 q S c_{kl} + p_1 p_2[D_{ik}(0) + D_{jl}(0)]\}}\end{aligned} \tag{D.15}$$

Just as in (D.8), the t dependence is completely lost. This is easily understood, since for $u=0$ the chains interpenetrate freely. The v contributions originate from correlations of the centers of mass due to the quenched random potentials. These correlations are necessarily time independent.

The u^2 term becomes

$$
\begin{aligned}
&g(\mathrm{q}, t-t')_{(2+2,\,u^2)} \\
&\quad = \frac{u^2}{2}\left(\frac{l^2}{4\pi\xi^2}\right)^{d/2} \int_{\mathrm{p}_1,\mathrm{p}_2} l^d (2\pi)^d \,\delta^d(\mathrm{q}-\mathrm{p}_1-\mathrm{p}_2) \\
&\qquad \times \sum_{i,j,k,l=1}^{N} \mathrm{e}^{-\mathrm{p}_1\mathrm{q}Sc_{ij}-\mathrm{p}_2\mathrm{q}Sc_{kl}-\mathrm{q}^2\Gamma|t-t'|} \\
&\qquad \times \Big[(1+\mathrm{q}^2\Gamma|t-t'|)\,\mathrm{e}^{\mathrm{p}_1\mathrm{p}_2[D_{ik}(0)+D_{jl}(0)]} \\
&\qquad + 2(\mathrm{p}_1\mathrm{q})(\mathrm{p}_2\mathrm{q}) \int_0^{|t-t'|} \Gamma\, d\tau\, \Gamma(|t-t'|-\tau)\,\mathrm{e}^{\mathrm{p}_1\mathrm{p}_2[D_{ik}(\tau)+D_{jl}(\tau)]} \Big]
\end{aligned}
\tag{D.16}
$$

The uv term is again the most complicated one. It finally "reduces" to

$$
\begin{aligned}
&g(\mathrm{q}, t-t')_{(2+2,\,vu)} + g(\mathrm{q}, t-t')_{(2+2,\,uv)} \\
&\quad = -uv\left(\frac{l^2}{4\pi\xi^2}\right)^{d/2} \int_{\mathrm{p}_1,\mathrm{p}_2} l^d (2\pi)^d \,\delta^d(\mathrm{q}-\mathrm{p}_1-\mathrm{p}_2) \\
&\qquad \times \sum_{i,j,k,l=1}^{N} \mathrm{e}^{-\mathrm{p}_1\mathrm{q}Sc_{ij}-\mathrm{p}_2\mathrm{q}Sc_{kl}-\mathrm{q}^2\Gamma|t-t'|} \\
&\qquad \times \Big[(1+\mathrm{q}^2\Gamma|t-t'|)\,\mathrm{e}^{\mathrm{p}_1\mathrm{p}_2(D_{ik}(0)+D_{jl}(0))} \\
&\qquad + \mathrm{q}^2 \int_0^{|t-t'|} \Gamma\, d\tau\, \Gamma(|t-t'|-\tau)\,(\mathrm{p}_1\mathrm{q}\,\mathrm{e}^{\mathrm{p}_1\mathrm{q}\Gamma\tau} + \mathrm{p}_2\mathrm{q}\,\mathrm{e}^{\mathrm{p}_2\mathrm{q}\Gamma\tau}) \\
&\qquad \times \mathrm{e}^{\mathrm{p}_1\mathrm{p}_2(D_{ik}(\tau)+D_{jl}(0))} \\
&\qquad + \frac{1}{2}\int_0^{|t-t'|} \Gamma^2\, d\tau\, d\tau'\, \theta(|t-t'|-\tau-\tau')[-1+\mathrm{q}^2\Gamma(|t-t'|-\tau-\tau')] \\
&\qquad \times [(\mathrm{p}_1\mathrm{q})^2\,\mathrm{e}^{\mathrm{p}_1\mathrm{q}\Gamma(\tau+\tau')} + (\mathrm{p}_2\mathrm{q})^2\,\mathrm{e}^{\mathrm{p}_2\mathrm{q}\Gamma(\tau+\tau')}] \\
&\qquad \times \mathrm{e}^{\mathrm{p}_1\mathrm{p}_2(D_{ik}(\tau)+D_{jl}(\tau'))} \Big]
\end{aligned}
\tag{D.17}
$$

Equations (D.15)–(D.17) yield the expression (5.13).

APPENDIX E. THE ϵ POLES OF $\mathscr{G}_{12}(q, t)$

Here we determine the ε poles of the one-loop expression $\mathscr{G}_{12}(q, t)_2$. This function consists of two parts [see (5.11)]

$$\mathscr{G}_{12}(q, t)_2 = g(q, t)_{(3+1)} + g(q, t)_{(2+2)}$$

$g(q, t)_{(3+1)}$ is given in Eq. (5.12) and will be analyzed first. Skipping prefactors, we start by considering the part

$$\mathscr{A}_1 = \int_0^S ds_i\, ds_j\, ds_k\, ds_l\, e^{-q^2 S c_{kl}}\, D_{ij}(0)^{-d/2} \times \left(1 - \exp \frac{q^2 (G_{ik}(0) - G_{jk}(0))^2}{4 D_{ij}(0)}\right) \tag{E.1}$$

In normalized segment coordinates $\mathbf{i}$ [cf. (5.16], with Eqs. (A.6), (A.9), and (A.10) inserted and with a Taylor expansion of the second exponential, $\mathscr{A}_1$ becomes

$$\mathscr{A}_1 = -S^{4-d/2} \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, d\mathbf{k}\, d\mathbf{l}\, e^{-Sq^2(1/6 + \mathbf{k}^2 + \mathbf{l}^2)} \times \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{Sq^2}{4}\right)^n |\mathbf{i} - \mathbf{j}|^{-2-n+\varepsilon/2} (\mathbf{i}^2 - \mathbf{j}^2 - |\mathbf{i} - \mathbf{k}| + |\mathbf{j} - \mathbf{k}|)^{2n} \tag{E.2}$$

The nth term of the expansion under the integral is proportional to $|\mathbf{i} - \mathbf{j}|^{-2+n+\varepsilon/2}$. The integral is a generic one of polymer statics just like (4.25). For $d = 4 - \varepsilon \leqslant 4$ all terms with $n \geqslant 2$ are regular. Only the $n = 1$ term contributes an ε pole:

$$\int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, |\mathbf{i} - \mathbf{j}|^{-3+\varepsilon/2} (\mathbf{i}^2 - \mathbf{j}^2 - |\mathbf{i} - \mathbf{k}| + |\mathbf{j} - \mathbf{k}|)^2 = \frac{4 + \mathcal{O}(\varepsilon)}{\varepsilon} \left(\frac{1}{3} + 4\mathbf{k}^2\right)$$

and the divergent part of $\mathscr{A}_1$ is explicitly

$$\mathscr{A}_1 = \frac{2 + \mathcal{O}(\varepsilon)}{\varepsilon} S^{\varepsilon/2} S^2 \int_{-1/2}^{1/2} d\mathbf{k}\, d\mathbf{l}\, e^{-Sq^2(1/6 + \mathbf{k}^2 + \mathbf{l}^2)} \times \left[-Sq^2 \left(\frac{1}{6} + \mathbf{k}^2 + \mathbf{l}^2\right)\right] \tag{E.3}$$

In (E.3) $2\mathbf{k}^2$ is replaced by $\mathbf{k}^2 + \mathbf{l}^2$ in the last factor, which is obviously allowed.

The next term to be analyzed in (5.12) is

$$\begin{aligned}\mathscr{A}_2 &= \int_0^S ds_i\, ds_j \int_0^t \Gamma^3\, d\tau\, d\tau'\, d\tau''\, \theta(t-\tau-\tau'-\tau'') \int \frac{d^d\mathrm{p}}{\pi^{d/2}} (\mathrm{pq})^2 \frac{\partial G_{jk}(\tau')}{\Gamma\, \partial\tau'} \\ &\quad \times \exp\{-\mathrm{p}^2 D_{ij}(\tau) - \mathrm{pq}[G_{ik}(\tau+\tau') - G_{jk}(\tau') - \Gamma\tau]\} \\ &= \frac{\mathrm{q}^2}{2} \int_0^S ds_i\, ds_j \int_0^t \Gamma\, d\tau\, \Gamma(t-\tau-\tau') \frac{\partial G_{jk}(\tau')}{\Gamma\, \partial\tau'} \\ &\quad \times \sum_{n=0}^{\infty} \frac{1+2n}{n!} \left(\frac{\mathrm{q}^2}{4}\right)^n D_{ij}(\tau)^{-3-n+\varepsilon/2} [G_{ik}(\tau+\tau') - G_{jk}(\tau') - \Gamma\tau]^{2n}\end{aligned} \tag{E.4}$$

In the last expression p is integrated out and the exponential is expanded. Divergencies in (E.4) can only originate from $D_{ij}(\tau)^{-3-n+\varepsilon/2}$. Due to the structure of $D_{ij}(\tau)$ as discussed in Appendix A, especially in Eqs. (A.40) and (A.44), a factor of $(2\hat{\gamma}\tau)^{1/2}$ can be factorized out of $D_{ij}(\tau)$ as the term causing divergencies. The rest of $D_{ij}(\tau)$ is finite, after the segments have been rescaled as

$$y = (s_i - s_j)/(2\hat{\gamma}\tau)^{1/2} \tag{E.5}$$

(The same has been done in Appendix B.) For the evaluation of (E.4) one furthermore needs the τ dependence of $[G_{ik}(\tau+\tau') - G_{jk}(\tau') - \Gamma\tau]$. For $\Gamma\tau/S \ll 1$ it takes the form

$$\begin{aligned}&[G_{ik}(\tau+\tau') - G_{jk}(\tau') - \Gamma\tau] \\ &\quad = [\{G_{ik}(\tau+\tau') - G_{ik}(\tau')\} + \{G_{ik}(\tau') - G_{jk}(\tau')\} - \Gamma\tau] \\ &\quad = S\left[\mathcal{O}\left(\frac{\Gamma\tau}{S}\right) + \mathcal{O}\left(y\left(\frac{2\Gamma\tau}{S}\right)^{1/2} - \frac{\Gamma\tau}{S}\right]\right. \\ &\quad = S\,\mathcal{O}\left(\frac{2\Gamma\tau}{S}\right)^{1/2} = \mathcal{O}(2\hat{\gamma}\tau)^{1/2}\end{aligned} \tag{E.6}$$

The full term

$$D_{ij}(\tau)^{-3-n+\varepsilon/2}[G_{ik}(\tau+\tau') - G_{jk}(\tau') - \Gamma\tau]^{2n}$$

in (E.4) for $\Gamma\tau/S \ll 1$ is thus proportional to $(2\hat{\gamma}\tau)^{3+n-\varepsilon/2)/2}$. Another factor of $(2\hat{\gamma}\tau)^{1/2}$ is gained by the substitution (E.5) of the segment coordinates. Therefore only the $n=0$ term in (E.4) contributes an ε pole and

$$\mathscr{A}_2 = \frac{q^2}{2}\int_0^S ds_j \int_0^t d\tau' \frac{G_{jk}(\tau')}{\partial\tau'} \int_0^{t-\tau'} \Gamma\, d\tau\, \Gamma(t-\tau-\tau')$$
$$\times \int_{-s_j/(2\hat{\gamma}\tau)^{1/2}}^{(S-s_j)/(2\hat{\gamma}\tau)^{1/2}} dy\, (2\hat{\gamma}\tau)^{(-2+\varepsilon/2)/2} F\left(y, \frac{2s_j}{(2\hat{\gamma}\tau)^{1/2}} + y, \frac{2S}{(2\hat{\gamma}\tau)^{1/2}}\right)^{-3+\varepsilon/2}$$
$$+ \mathrm{reg}(\varepsilon) \tag{E.7}$$

The evaluation of the ε pole proceeds now analogously to (B.5) and yields as a first step

$$\mathscr{A}_2 = \frac{q^2}{2}\int_0^S ds_j \int_0^t d\tau' \frac{\partial G_{jk}(\tau')}{\partial\tau'} \frac{\Gamma(t-\tau')}{S} \frac{4I+\mathcal{O}(\varepsilon)}{\varepsilon} (\min\{S^2, 2\hat{\gamma}t\})^{\varepsilon/4} \tag{E.8}$$

Making use of (4.12), one finds

$$\int_0^S ds_j \frac{\partial G_{jk}(\tau')}{\partial\tau'} = -S\,\Gamma\, e^{-\Gamma\tau'/\xi^2} \tag{E.9}$$

and

$$\mathscr{A}_2 = -q^2(\Gamma t)^2 \left(1 + \mathcal{O}\left(\frac{\Gamma t}{\xi^2}\right)\right) \frac{I+\mathcal{O}(\varepsilon)}{\varepsilon} (\min\{S^2, 2\hat{\gamma}t\})^{\varepsilon/4} \tag{E.10}$$

Using the equivalence (E.1) = (E.3) and (E.4) = (E.10), we derive Eq. (5.14) from (5.12).

We now proceed to the analysis of the $2+2$ diagrams as given in (5.13). There are four terms, of which only the first contributes an ε pole.

This first term $g(q, t)_{(2+2,1)}$ reads, after integrating over p_1 and p_2 and in normalized segment coordinates,

$$g(q, t)_{(2+2,1)}$$
$$= \xi^{-d} \frac{1}{8} [\hat{v}^2 + (\hat{u} - 2\hat{v})\,\hat{u}\,(1 + q^2\Gamma t)\, e^{-q^2\Gamma t}]$$
$$\times S^{4-d/2} \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, d\mathbf{k}\, d\mathbf{l}$$
$$\times e^{-Sq^2(1/6 + \mathbf{i}^2 + \mathbf{j}^2)} (|\mathbf{i}-\mathbf{k}| + |\mathbf{j}-\mathbf{l}|)^{-d/2}$$
$$\times \exp \frac{Sq^2(|\mathbf{i}-\mathbf{k}| + |\mathbf{j}-\mathbf{l}| + \mathbf{k}^2 - \mathbf{i}^2 + \mathbf{l}^2 - \mathbf{j}^2)^2}{4(|\mathbf{i}-\mathbf{k}| + |\mathbf{j}-\mathbf{l}|)} \tag{E.11}$$

The divergence appears for $d \leqslant 4$ if simultaneously $|\mathbf{i}-\mathbf{k}| \to 0$ and $|\mathbf{j}-\mathbf{l}| \to 0$. The expression in the last exponential is of order $S\mathrm{q}^2 \mathcal{O}(|\mathbf{i}-\mathbf{k}|+|\mathbf{j}-\mathbf{l}|)$. Again the leading term of the Taylor expansion of the exponential contributes the leading singularity. This leading term is here unity. The essential integration for deriving the ε pole is

$$\int_{-1/2}^{1/2} d\mathbf{k}\, d\mathbf{l}\, (|\mathbf{i}-\mathbf{k}|+|\mathbf{j}-\mathbf{l}|)^{-2+\varepsilon/2} = \frac{8+\mathcal{O}(\varepsilon)}{\varepsilon} \tag{E.12}$$

and

$$g(\mathrm{q}, t)_{(2+2,1)} = \xi^{-d}[\hat{v}^2 + (\hat{u}-2\hat{v})\,\hat{u}\,(1+\mathrm{q}^2\Gamma t)\,\mathrm{e}^{-\mathrm{q}^2\Gamma t}] \times S^2 \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, \mathrm{e}^{-S\mathrm{q}^2(1/6+\mathbf{i}^2+\mathbf{j}^2)} \frac{S^{\varepsilon/2}+\mathcal{O}(\varepsilon)}{\varepsilon} \tag{E.13}$$

The second term on a level equivalent to (E.11) reads

$$\begin{aligned} g(\mathrm{q}, t)_{(2+2,2)} = \xi^{-d} \frac{1}{8} \int_{-1/2}^{1/2} & d\mathbf{k}\, d\mathbf{l}\, \mathrm{e}^{-S\mathrm{q}^2(1/6+\mathbf{k}^2+\mathbf{l}^2)} \\ & \times 2\hat{u}^2\, \mathrm{e}^{-\mathrm{q}^2\Gamma t} \int_0^t d\tau\, (t-\tau) \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, \frac{\mathrm{q}^2}{2} \\ & \times [D_{ik}(\tau)+D_{jl}(\tau)]^{-d/2-1} \left(B\mathrm{q}^2 - 1 - \frac{B^2\mathrm{q}^2}{D_{ik}(\tau)+D_{jl}(\tau)} \right) \\ & \times \exp \frac{B^2\mathrm{q}^2}{D_{ik}(\tau)+D_{jl}(\tau)} \end{aligned} \tag{E.14}$$

with

$$B = D_{ik}(\tau) + D_{jl}(\tau) + \mathbf{k}^2 - \mathbf{i}^2 + \mathbf{l}^2 - \mathbf{j}^2$$

Again the zeros of B are of the order of $D_{ik}(\tau)+D_{jl}(\tau)$. This implies that again the most divergent term is given by the lowest order of the Taylor expansion of the last exponential. The divergence of this term is due to the integral

$$\begin{aligned} & \int_0^t d\tau\, (t-\tau) \int_{-1/2}^{1/2} d\mathbf{i}\, d\mathbf{j}\, [D_{ik}(\tau)+D_{jl}(\tau)]^{-1-d/2} \\ & = \int_0^t d\tau\, (t-\tau) \int_{(-s_k+S/2)/(2\hat{\gamma}\tau)^{1/2}}^{(-s_k+S/2)/(2\hat{\gamma}\tau)^{1/2}} \frac{dx}{S} \int_{(-s_l+S/2)/(2\hat{\gamma}\tau)^{1/2}}^{(-s_l+S/2)/(2\hat{\gamma}\tau)^{1/2}} \frac{dy}{S}\, (2\hat{\gamma}\tau)^{(-1+\varepsilon/2)/2} \\ & \quad \times \left[F\left(x, \frac{2s_k}{(2\hat{\gamma}\tau)^{1/2}} + x, \frac{2S}{(2\hat{\gamma}\tau^{1/2}}\right) + F\left(y, \frac{2s_l}{(2\hat{\gamma}\tau)^{1/2}} + y, \frac{2S}{(2\hat{\gamma}\tau)^{1/2}}\right) \right]^{-3+\varepsilon/2} \end{aligned} \tag{E.15}$$

A singularity can only result from the τ integration. It only appears for $d \geqslant 6$.

The treatment of the next term is analogous. The most divergent term contains

$$(D_{ik}(\tau) + D_{jl}(0))^{-2+\varepsilon/2}$$

Divergencies only appear for $d \geqslant 8$.

The last term finally contains

$$(D_{ik}(\tau) + D_{jl}(\tau'))^{-3+\varepsilon/2}$$

It diverges only for $d \geqslant 10$.

APPENDIX F. THE ϵ POLES OF $\mathscr{G}_{\text{int}}(i, j, \mathbf{q}, t)$

In the expression (6.6) there are three terms to be analyzed. The first one is

$$\begin{aligned}\mathscr{A}_1 &= \int_0^S ds_k\, ds_m\, |s_k - s_m|^{-d/2} \left(\exp \frac{\mathbf{q}^2 \mathscr{B}_{ijkm}(0,0,t)^2}{4\,|s_k - s_m|} - 1 \right) \\ &= \int_0^S ds_k \int_{-s_k}^{S-s_k} dx \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\mathbf{q}^2}{4} \right)^n |x|^{-n-d/2}\, \mathscr{B}_{ijk(k+x/t^2)}(0,0,t)^{2n} \quad \text{(F.1)}\end{aligned}$$

$\mathscr{B}_{ijkm}(0, \tau_1, \tau_2)$ can be written as

$$\mathscr{B}_{ijkm}(0, \tau_1, \tau_2) = b_i(k, m, \tau_1) - b_j(k, m, \tau_2) \quad \text{(F.2)}$$

with

$$b_i(k, m, \tau) = D_{ik}(\tau) - D_{im}(\tau)$$

For $\tau = 0$

$$b_i(k, m, 0) = |s_i - s_k| - |s_i - s_m| = \pm \mathscr{C}(s_k - s_m) \quad \text{(F.3)}$$

For $\tau \neq 0$ one derives by means of (A.28), (A.33), (A.39), and (A.24), that

$$\begin{aligned}\frac{b_i(k, m, \tau)}{(2\hat{\gamma}\tau)^{1/2}} &= |y|^{*} - |y + y_2| \\ &\quad + \sum_{\nu=-\infty}^{\infty} [\, g(y + \nu\mu) - g(y + y_2 + \nu\mu) \\ &\quad + g(z + \nu\mu) - g(z - y_2 + \nu\mu)\,]\end{aligned}$$

$$= \sum_{\nu=-\infty}^{\infty} [f(y+\nu\mu) - f(y+y_2+\nu\mu) + f(z+\nu\mu) - f(z-y_2+\nu\mu)]$$

$$= y_2 \sum_{\nu=-\infty}^{\infty} [-\mathrm{erf}(y+\nu\mu) + \mathrm{erf}(z+\nu\mu)] + \mathcal{O}(y_2^2)$$

$$= y_2 \frac{\partial D_{ik}(\tau)}{\partial s_k} + \mathcal{O}(y_2^2) \tag{F.4}$$

with

$$y = \frac{s_i - s_k}{(2\hat{\gamma}\tau)^{1/2}}, \quad y_2 = \frac{s_k - s_m}{(2\hat{\gamma}\tau)^{1/2}}, \quad z = \frac{s_i + s_k}{(2\hat{\gamma}\tau)^{1/2}}, \quad \mu = \frac{2S}{(2\hat{\gamma}\tau)^{1/2}} \tag{F.5}$$

$b_i(k, m, \tau)/(2\hat{\gamma}\tau)^{1/2}$ is expandable in y_2 to all orders, because

$$f(x+y_2) = f(x) + y_2 \,\mathrm{erf}\, x + \sum_{n=2}^{\infty} \frac{(-y_2)^2}{n!} H_{n-2}(x) \frac{2}{\sqrt{\pi}} \, \mathrm{e}^{-x^2} \tag{F.6}$$

with the Hermitian polynomials $H_n(x)$.

For the present calculation it makes no difference whether $b_i(k, m, \tau)$ is assumed to be expandable in $(s_k - s_m)$ or whether actually $b_i(k, m, \tau)/(2\hat{\gamma}\tau)^{1/2}$ is expandable in y_2. The distinction, however, becomes crucial in higher orders of perturbation theory. In any case we conclude that $\mathscr{B}_{ijk(k+x/l^2)}(0, 0, t) = \mathcal{O}(x)$. Thus in $\mathscr{A}_1$, (F.1), only the $n=1$ term contributes a singularity. It is

$$\mathscr{A}_1 = [1 + \mathcal{O}(\varepsilon)] \int_0^S ds_k \int_{-s_k}^{S-s_k} dx \, \frac{\mathrm{q}^2}{4} |x|^{1-d/2}$$

$$\times \left\{ \frac{|s_i - s_k - x| - |s_i - s_k|}{x} - \frac{\partial D_{jk}(t)}{\partial s_k} \left[1 + \mathcal{O}\left(\frac{x}{(2\hat{\gamma}\tau)^{1/2}} \right) \right] \right\}^2$$

$$= \frac{1 + \mathcal{O}(\varepsilon)}{\varepsilon} S^{\varepsilon/2} \mathrm{q}^2 \int_0^S ds_k \left(\frac{\partial (D_{ik}(0) - D_{jk}(t))}{\partial s_k} \right)^2 \tag{F.7}$$

The following identity can be derived from (A.26) and orthogonality relations of sin and cos:

$$\int_0^S ds_k \left(\frac{\partial (D_{ik}(\tau) - D_{jk}(\tau'))}{\partial s_k} \right)^2 = 4D_{ij}(|\tau| + |\tau'|) - 2D_{ii}(2\tau) - 2D_{jj}(2\tau') \tag{F.8}$$

Accordingly, $\mathscr{A}_1$ becomes

$$\mathscr{A}_1 = \frac{1 + \mathcal{O}(\varepsilon)}{\varepsilon} S^{\varepsilon/2} \mathrm{q}^2 [4D_{ij}(t) - 2D_{jj}(2t)] \tag{F.9}$$

The second expression to be analyzed in (6.6) is

$$\mathscr{A}_2 = \int_0^S ds_k\, ds_m\, |s_k - s_m|^{-d/2} \int_0^t d\tau\, \frac{\partial}{\partial t} \exp \frac{q^2 \mathscr{B}_{ijkm}(0, \tau, t-\tau)^2}{4\,|s_k - s_m|}$$

$$= 2 \int_0^S ds_k \int_{-s_k}^{S-s_k} dx \int_0^t d\tau \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{q^2}{4}\right)^{n+1} |x|^{n+1-d/2}$$

$$\times \left(\frac{\mathscr{B}_{ijk(k+x/l^2)}(0, \tau, t-\tau)}{x} \right)^{2n+1} \frac{\partial \mathscr{B}_{ijk(k+x/l^2)}(0, \tau, t-\tau)}{x\, \partial t} \tag{F.10}$$

The discussion of $\mathscr{B}_{ijkm}(0, \tau_1, \tau_2)$ applies here again. Only the $n=0$ term contributes a singularity and

$$\mathscr{A}_2 = [1 + \mathcal{O}(\varepsilon)] \frac{q^2}{4} \int_0^S ds_k \int_{-s_k}^{S-s_k} dx \int_0^t d\tau\, |x|^{1-d/2}$$

$$\times \frac{\partial}{\partial t} \left(\frac{b_i(k, k+x/l^2, \tau) - b_j(k, k+x/l^2, t-\tau)}{x} \right)^2$$

$$= \frac{1 + \mathcal{O}(\varepsilon)}{\varepsilon} S^{\varepsilon/2} q^2 \int_0^S ds_k \int_0^t d\tau \frac{\partial}{\partial t} \left(\frac{\partial (D_{ik}(\tau) - D_{jk}(t-\tau))}{\partial s_k} \right)^2 \tag{F.11}$$

Insertion of (F.8) and partial integrations in time yield

$$\mathscr{A}_2 = \frac{1 + \mathcal{O}(\varepsilon)}{\varepsilon} S^{\varepsilon/2} q^2 \left(4t \frac{\partial D_{ij}(t)}{\partial t} - 2D_{jj}(2t) \right) \tag{F.12}$$

The last term to be discussed in (6.6) is

$$\mathscr{A}_3 = \int_0^S ds_k\, ds_l\, q^2 \int_0^t d\tau\, d\tau'\, \theta(t-\tau-\tau')\, D_{kl}(\tau)^{-3+\varepsilon/2}$$

$$\times \frac{\partial D_{jk}(\tau')}{\partial \tau'} \frac{\partial D_{il}(t-\tau-\tau')}{\partial (t-\tau-\tau')} \sum_{n=0}^{\infty} \frac{2n+1}{n!} \left(\frac{q^2 \mathscr{B}_{ijkl}(\tau, \tau', t-\tau')^2}{4 D_{kl}(\tau)} \right)^n \tag{F.13}$$

where again the last exponential has been expanded. Singularities here come from $D_{kl}(\tau)$ and more specifically from factors of $(2\hat{\gamma}\tau)^{1/2}$, which can be extracted from $D_{kl}(\tau)$ similarity as, e.g., in (B.4). Then $s_k - s_l$ again has to be rescaled as $y = (s_k - s_l)/(2\hat{\gamma}\tau)^{1/2}$. In an argument similar to (E.6) one can convince oneself, that $\mathscr{B}_{ijkl}(\tau, \tau', t-\tau') = \mathcal{O}((2\hat{\gamma}\tau)^{1/2})$. Therefore again only the $n=0$ term contributes a singularity:

$$
\begin{aligned}
&\mathscr{A}_3(1+\mathcal{O}(\varepsilon)) \\
&\quad = \mathrm{q}^2 \int_0^S ds_k\, ds_l \int_0^t d\tau' \int_0^{t-\tau'} d\tau \\
&\qquad \times D_{kl}(\tau)^{-3+\varepsilon/2} \frac{\partial D_{jk}(\tau')}{\partial \tau'} \frac{\partial D_{il}(t-\tau-\tau')}{\partial (t-\tau-\tau')} \\
&\quad = \mathrm{q}^2 \int_0^S ds_k \int_0^t d\tau' \int_0^{t-\tau'} d\tau \int_{-s_k/(2\hat{\gamma}\tau)^{1/2}}^{(S-s_k)/(2\hat{\gamma}\tau)^{1/2}} dy\, (2\hat{\gamma}\tau)^{(-2+\varepsilon/2)/2} \\
&\qquad \times F\left(y, \frac{2s_k}{(2\hat{\gamma}\tau)^{1/2}} + y, \frac{2S}{(2\hat{\gamma}\tau)^{1/2}}\right)^{-3+\varepsilon/2} \\
&\qquad \times \frac{\partial D_{jk}(\tau')}{\partial \tau'} \frac{\partial D_{i[k+y(2\hat{\gamma}\tau)^{1/2}/l^2]}(t-\tau-\tau')}{\partial (t-\tau-\tau')} \\
&\quad = \frac{4I+\mathcal{O}(\varepsilon)}{\varepsilon} (\min\{S^2, 2\hat{\gamma}t\})^{\varepsilon/4} \frac{\mathrm{q}^2}{\hat{\gamma}} \\
&\qquad \times \int_0^S ds_k \int_0^t d\tau' \frac{\partial D_{jk}(\tau')}{\partial \tau'} \frac{\partial D_{ik}(t-\tau')}{\partial (t-\tau')}
\end{aligned}
\tag{F.14}
$$

From the representation (A.26) of $D_{ij}(t)$ one derives that

$$
\int_0^S \frac{ds_k}{\hat{\gamma}} \frac{\partial D_{jk}(\tau')}{\partial \tau'} \frac{\partial D_{ik}(\tau)}{\partial \tau} = \operatorname{sgn}(\tau) \operatorname{sgn}(\tau') \frac{\partial D_{ij}(|\tau|+|\tau'|)}{\partial (|\tau|+|\tau'|)} \tag{F.15}
$$

Insertion into (F.14) finally yields

$$
\mathscr{A}_3 = \frac{4I+\mathcal{O}(\varepsilon)}{\varepsilon} (\min\{S^2, 2\hat{\gamma}t\})^{\varepsilon/4}\, \mathrm{q}^2 t \frac{\partial D_{ij}(t)}{\partial t} \tag{F.16}
$$

Insertion of these identities into (6.6) finally results in

$$
\begin{aligned}
\mathscr{G}_{\mathrm{int}}(i,j,\mathrm{q},t)_{\mathrm{l}} = \mathrm{e}^{-\mathrm{q}^2 D_{ij}(t)} \Bigg[&\frac{\hat{w} S^{\varepsilon/2} + \mathcal{O}(\varepsilon)}{\varepsilon} \left(-1 + \frac{\partial}{\partial \ln t}\right) \\
&+ \frac{I\hat{v}\,(\min\{S^2, (2\hat{\gamma}t)^{1/2}\})^{\varepsilon/4} + \mathcal{O}(\varepsilon)}{\varepsilon} \frac{\partial}{\partial \ln t} \Bigg] \mathrm{q}^2 D_{ij}(t)
\end{aligned}
\tag{F.17}
$$

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